KRISHNA SUPPORT STUDY MATERIAL

XII Maths

Support Material, Key Points, HOTS and VBQ

STUDY MATERIAL

SUBJECT: MATHEMATICS

CLASS :XII

INDEX

Topic wise Analysis of Examples and Questions

NCERT TEXT BOOK

Chapters	Concepts	Number of Questions for revision		Total
		Questions From Solved Examples	Questions From Exercise	
01	**Relations & Functions**	**15**	**25**	**40**
02	**Inverse Trigonometric Functions**	**05**	**09**	**14**
03	**Matrices & Determinants**	**06**	**25**	**31**
04	**Continuity& Differentiability**	**08**	**21**	**29**
05	**Application of Derivative**	**06**	**14**	**20**
06	**Indefinite Integrals**	**17**	**45**	**62**
07	**Applications of Integration**	**05**	**09**	**14**
08	**Differential Equations**	**07**	**19**	**26**
09	**Vector Algebra**	**07**	**18**	**25**
10	**Three Dimensional Geometry**	**07**	**12**	**19**
11	**Linear Programming**	**09**	**12**	**21**
12	**Probability**	**19**	**27**	**46**
	TOTAL	**111**	**236**	**347**

Detail of the concepts to be mastered by every child of class XII with exercises and examples of NCERT Text Book.

SYMBOLS USED

*** : Important Questions, ** :Very Important Questions,**

***** : Very-Very Important Questions**

S.No	Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
1	***Relations & Functions***	(i) .Domain , Co-domain & Range of a relation	*	(Previous Knowledge)
		(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
		(iii).One-one , onto & inverse ofa function	***	Ex 1.2 Q.No- 7,9
		(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
		(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11
2	***Inverse Trigonometric Functions***	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
		(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 QNo- 7,13, 15 Misc Ex Q.No.9,10,11,12
3	***Matrices & Determinants***	(i) Order, Addition, Multiplication and transpose of matrices	***	Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
		(ii) Cofactors &Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
		(iii)Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4,Q.No4,5,8,12,15
		(iv)To find difference between $\vert A\vert$, $\vert adj\ A\vert$, $\vert kA\vert$, $\vert A.adjA\vert$	*	Ex 4.1 –Q.No 3,4,7,8
		(v) Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18
4	***Continuity& Differentiability***	(i).Limit of a function	*	
		(ii).Continuity	***	Ex 5.1 Q.No- 21, 26,30
		(iii).Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
		(iv).Logrithmic Differentiation	***	Ex 5.5 Q.No- 6,9,10,15
		(v) Parametric Differentiation	***	Ex 5.6 Q.No- 7,8,10,11
		(vi). Second order derivatives	***	Ex 5.7 Q.No- 14,16,17

		(vii). M. V.Th	**	Ex 5.8 Q.No- 3,4
5	***Application of Derivative.***	(i).Rate of change	*	Example 5Ex 6.1 Q.No- 9,11
		(ii).Increasing & decreasing functions	***	Ex 6.2 ,Q.No- 6 Example 12,13
		(iii).Tangents & normal	**	Ex 6.3 ,Q.No- 5,8,13,15,23
		(iv).Approximations	*	Ex 6.4,Q.No- 1,3
		(v) Maxima & Minima	***	Ex 6.5, Q.No- 8,22,23,25 Example 35,36,37
6	***Indefinite Integrals***	(i) Integration by substitution	*	Exp 5&6 Page301,303
		(ii) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
		(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	***	Edition Exp 8, 9, 10 Page 311,312Exercise 7.4 Q 3,4,8,9,13&23
		(iv) Integration using Partial Fraction	***	EditionExp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
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		(iii)Unit vector	* *	Ex 6,8 Pg 436
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		(v)Dot product of two vectors	**	Q6 ,13 Pg445
		(vi)Projection of a vector	* * *	Ex 16 Pg 445
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		(viii)Area of a triangle	*	Q 9 Pg 454
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10	***Three Dimensional Geometry***	(i)Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
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12	***Probability***	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1
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		(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
		(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
		(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
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TOPIC 1
RELATIONS & FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XII Ed. 2007
Relations & Functions	(i).Domain , Co domain & Range of a relation	*	(Previous Knowledge)
	(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
	(iii).One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
	(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
	(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

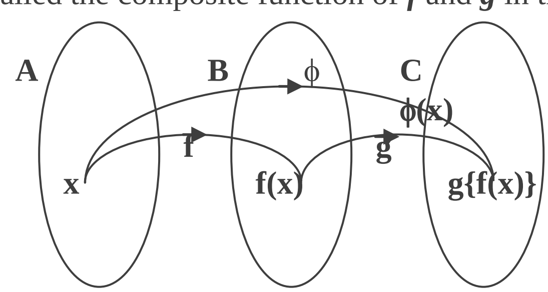
SOME IMPORTANT RESULTS/CONCEPTS

** A **relation** R in a set A is called
(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,
(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
(iii)*transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.
** Equivalence Relation : R is equivalence if it is reflexive, symmetric and transitive.

** **Function** :A relation $f : A \to B$ is said to be a function if every element of A is correlatedto unique element in B.
* A is domain
* B is codomain
* For any x element $x \in A$, function f correlates it to an element in B, which is denoted by f(x)and is called image of x under f . Again if y= f(x), then x is called as pre-image of y.
* Range = $\{f(x) \mid x \in A\}$. Range $\subseteq$ Codomain
* The largest possible domain of a function is called domain of definition.

Composite function :

Let two functions be defined as $f : A \to B$ and $g : B \to C$. Then we can define a function $\phi : A \to C$ by setting $\phi(x) = g\{f(x)\}$ where $x \in A$, $f(x) \in B$, $g\{f(x)\} \in C$. This function $\phi : A \to C$ is called the composite function of f and g in that order and we write. $\phi = g o f$.



**** Different type of functions** : Let ***f*** : A $\rightarrow$B be a function.

*** *f*** is **one to one (injective) mapping**, if any two different elements in A is always correlated to different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$or, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

*** *f*** is **many one mapping,** if $\exists$ at least two elements in A such that their images are same.

*** *f*** is **onto mapping** (subjective), if each element in B is having at least one preimage.

****f*** is **into mapping** if range $\subseteq$ codomain.

*** *f*** is **bijective mapping** if it is both *one to one and onto*.

**** Binary operation :** A binary operation * on a set A is a function * : A × A → A. We denote *(*a*, *b*) by *a* **b*.

* A binary operation '*' on A is a rule that associates with every ordered pair (a, b) of A x A a unique element a *b.

* An operation '*' on a is said to be commutative iff a * b = b * a $\forall$ a, b $\in$ A.

* An operation '*' on a is said to be associative iff (a * b) * c = a * (b * c) $\forall$ a, b, c $\in$ A.

* Given a binary operation * : A × A → A, an element $e \in$ A, if it exists, is called ***identity*** for the operation *, if $a * e = a = e * a$, $\forall a \in$ A.

* Given a binary operation * : A × A → A with the identity element *e* in A, an element $a \in$ A is said to be ***invertible*** with respect to the operation*, if there exists an element *b* in A such that $a * b = e = b * a$ and *b* is called the *inverse of a* and is denoted by a^{-1}.

ASSIGNMENTS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b € A. Write the domain, range & co-domain.
2. Define a relation R on the set **N** of natural numbers by

   R={(*x* , *y*) : *y* = *x* +7, *x* is a natural number lesst han 4 ; *x*, *y* ∈ **N**}.

   Write down the domain and the range.

2. *Types of relations*

LEVEL II

1. Let R be the relation in the set N given by R = {(a , b)| a = b – 2 , b > 6}
   Whether the relation is reflexive or not ?justify your answer.
2. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b $\in$ N} is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) $\in$R iff 1+ ab> 0 $\forall$ a,b$\in$R.
   (a) Show that R is symmetric.
   (b) Show that R is reflexive.
   (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x , y)| x – 3y = 0} on A ={1, 2, 3……….13, 14}.

LEVEL III

1. Show that the relation R on A ,A = { x| x $\in$ Z , $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.
2. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.
3. Show that the relation R in the set A of all polygons as:
   R ={(P_1,P_2), P_1& P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?
4. Show that the relation R on A ,A = { x| x $\in$ Z , $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.
5. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c ,d) iff a + d = b + c}. Show that R is an equivalence relation. [**CBSE 2010]**
6. Let A = Set of all triangles in a plane and R is defined by R={(T_1,T_2) : T_1,$T_2 \in$ A & $T_1 \sim T_2$ }
   Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5;
   T_2 with size 5,12,13; T_3 with side 6,8,10; Which of the pairs are related?

(iii)One-one , onto & inverse of a function

LEVEL I

1. If f(x) = $x^2 - x^{-2}$, then find f(1/x).
2 Show that the function f: R$\rightarrow$R defined by f(x)=x^2 is neither one-one nor onto.

3 Show that the function f: N$\rightarrow$N given by f(x)=2x is one-one but not onto.

4 Show that the signum function f: R$\rightarrow$R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

5 Let A = {-1,0,1} and B = {0,1}. State whether the function f : A $\rightarrow$ B defined by f(x) = x^2 isbijective .

6. Let f(x) = $\frac{x-1}{x+1}$, x$\neq$ -1,then find $f^{-1}(x)$

LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4),(2,5), (3,6)} be a function from A to B. State whether f is one-one or not. [**CBSE2011]**
2. If f : R$\rightarrow$R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function . Find $f^{-1}(x)$.
3. Write the number of all one-one functions on the set A={a, b, c} to itself.
4. Show that function f :R$\rightarrow$R defined by f(x)= $7 - 2x^3$ for all x $\in$R is bijective.
5. If f: R$\rightarrow$R is defined by f(x)= $\frac{3x+5}{2}$. Find f^{-1}.

LEVEL III

1. Show that the function f: R→R defined by $f(x) = \frac{2x-1}{3}$.$x \in$ R is one- one & onto function. Also find the f^{-1}.
2. Consider a function f :R_+→[-5, ∞) defined $f(x) = 9x^2 + 6x - 5$. Show that f is invertible & $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$, where $R_+ = (0,\infty)$.
3. Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y) = \frac{-3}{4}$.
4. Show that f: R→R defined by $f(x)= x^3+4$ is one-one, onto. Show that $f^{-1}(x)=(x-4)^{1/3}$.
5. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one one onto and hence find f^{-1}. **[CBSE2012]**
6. Show that $f : N \to N$ defined by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one one onto. **[CBSE2012]**

(iv) *Composition of functions*

LEVEL I

1. If $f(x) = e^{2x}$ and $g(x) = \log \sqrt{x}$, x > 0, find
   (a) (f + g)(x) (b) (f .g)(x) (c) f o g (x) (d) g o f (x).
2. If $f(x) = \frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = -\ f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

LEVEL II

1. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [x] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).
2. Let $f(x) = \frac{x-1}{x+1}$. Then find f(f(x))
3. If $y = f(x) = \frac{3x+4}{5x-3}$, then find (fof)(x) i.e. f(y)
4. Let f : **R** →**R** be defined as f(x) = 10x +7.Find the function g : **R**→**R**such that $g \circ f(x) = f \circ g(x) = I_R$ **[CBSE2011]**
5. If f : **R** →**R** be defined as f(x) = $\left(3-x^3\right)^{\frac{1}{3}}$, then find $f \circ f(x)$. **[CBSE2010]**
6. Let f :R→R& g : R→R be defined as $f(x) = x^2$, g(x) = 2x – 3 . Find fog(x).

(v)Binary Operations

LEVEL I

1. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
2. Let *be the binary on N given by a*b =HCF of {a ,b} , a,b$\in$N. Find 20*16.
3. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.

   Write the identity of *, if any.
4. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$

   Then find the value of 2 * 4.

<u>LEVEL 2</u>

1. Let A= N×N & * be the binary operation on A defined by (a ,b) * (c ,d) = (a+c, b+d)

   Show that * is (a) Commutative (b) Associative (c) Find identity for * on A, if any.

2. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).

   Find: (i) the identity element of A (ii) the invertible element of A.
3. Examine which of the following is a binary operation

   (i) $a * b = \frac{a+b}{2}$; a, b$\in$ N (ii) $a*b = \frac{a+b}{2}$ a, b$\in$ Q

   For binary operation check commutative & associative law.

<u>LEVEL 3</u>

1.Let A= N×N & * be a binary operation on A defined by (a , b) × (c , d) = (ac , bd)

$\forall$ (a , b),(c , d) $\in$N×N (i) Find (2,3) * (4,1)

(ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal

(iii) Show that * is commutative & associative on A.

2. Define a binary operation * on the set {0,1,2,3,4,5} as $a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & a+b \geq 6 \end{cases}$

Show that zero in the identity for this operation & each element of the set is invertible with 6 – a being the inverse of a. **[CBSE2011]**

3. Consider the binary operations $*$:$\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$and o : $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ defined as $a * b = |a - b|$ and a o b = a, $\forall$a, b $\in$**R**. Show that $*$is commutative but not associative, o is associative but not commutative. **[CBSE2012]**

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

2. Show that each of the relation R in the set A = {x $\in$ **Z** : $0 \leq x \leq 12$}, given by R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

3. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

4. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\to$ B defined by f (x) = $\left(\frac{x-2}{x-3}\right)$.

Is f one-one and onto? Justify your answer.

6. Consider f :**R+** $\to$ [– 5, ∞) given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible and find f^{-1}.

7. On R – {1} a binary operation '$*$' is defined as $a * b = a + b - ab$. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'.Also prove that every element of R – {1} is invertible.

8. If A = $Q \times Q$ and '$*$' be a binary operation defined by $(a, b) * (c, d) = (ac, b + ad)$, for (a, b), (c, d) $\in$ A .Then with respect to '$*$' on A

(i) examine whether '$*$' is commutative & associative

(i) find the identity element in A,

(ii) find the invertible elements of A.

TOPIC 2
INVERSE TRIGONOMETRIC FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Inverse Trigonometric Functions	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
	(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 Q No- 7,13, 15 Misc Ex Q.No. 9,10,11,12

SOME IMPORTANT RESULTS/CONCEPTS

* Domain & Range of the Inverse Trigonometric Function :

	Functions	Domain	Range (Principal value Branch)
i.	$\sin^{-1}$:	$[-1,1]$	$[-\pi/2, \pi/2]$
ii.	$\cos^{-1}$:	$[-1,1]$	$[0,\pi]$
iii.	$\operatorname{cosec}^{-1}$:	$R-(-1,1)$	$[-\pi/2,\pi/2]-\{0\}$
iv.	$\sec^{-1}$:	$R-(-1,1)$	$[0,\pi]-\{\pi/2\}$
v.	$\tan^{-1}$:	R	$(-\pi/2,\pi/2)$
vi.	$\cot^{-1}$:	R	$(0,\pi)$

* Properties of Inverse Trigonometric Function

1. (i) $\sin^{-1}(\sin x)=x$ & $\sin(\sin^{-1}x)=x$
   ii. $\cos^{-1}(\cos x)=x$ & $\cos(\cos^{-1}x)=x$
   iii. $\tan^{-1}(\tan x)=x$ & $\tan(\tan^{-1}x)=x$
   iv. $\cot^{-1}(\cot x)=x$ & $\cot(\cot^{-1}x)=x$
   v. $\sec^{-1}(\sec x)=x$ & $\sec(\sec^{-1}x)=x$
   vi. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$ & $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x$

2. i. $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ & $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$
   ii. $\cos^{-1}x=\sec^{-1}\frac{1}{x}$ & $\sec^{-1}x=\cos^{-1}\frac{1}{x}$
   iii. $\tan^{-1}x=\cot^{-1}\frac{1}{x}$ & $\cot^{-1}x=\tan^{-1}\frac{1}{x}$

3. i $\sin^{-1}(-x)=-\sin^{-1}x$
   ii. $\tan^{-1}(-x)=-\tan^{-1}x$
   iii. $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$
   iv $\cos^{-1}(-x)=\pi-\cos^{-1}x$
   v $\sec^{-1}(-x)=\pi-\sec^{-1}x$
   vi $\cot^{-1}(-x)=\pi-\cot^{-1}x$

4. i $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$
   ii. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$
   iii. $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$

5. $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

6. $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy<1$

$\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy>1$

$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $xy>-1$

ASSIGNMENTS

(i). Principal value branch Table

LEVEL I

Write the principal value of the following :

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(-\sqrt{3}\right)$

4. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

LEVEL II

Write the principal value of the following :

1. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[CBSE 2011]**

2. $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii). Properties of Inverse Trigonometric Functions

LEVEL I

1. Evaluate $\cot[\tan^{-1}a+\cot^{-1}a]$

2. Prove $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right)$

3. Find x if $\sec^{-1}\left(\sqrt{2}\right)+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$

LEVEL II

1. Write the following in simplest form : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x\neq 0$

2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

3. Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

4. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$ **[CBSE 2011]**

5. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$ **[CBSE 2012]**

LEVEL III

1. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ **[CBSE 2011]**

3. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

4. Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

5.Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ **[CBSE 2012]**

Questions for self evaluation

1. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\qquad x\in\left[-\frac{1}{\sqrt{2}},1\right]$

3. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

4. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

6. Write in the simplest form $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

7. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

8. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

TOPIC 3
MATRICES & DETERMINANTS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Matrices & Determinants	(i) Order, Addition, Multiplication and transpose of matrices	***	Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
	(ii) Cofactors &Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
	(iii)Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4–Q.No 4,5,8,12,15
	(iv)To find difference between $\lvert A\rvert$, $\lvert \text{adj } A\rvert$, $\lvert kA\rvert$, $\lvert A.\text{adj}A\rvert$	*	Ex 4.1 –Q.No 3,4,7,8
	(v) Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18

SOME IMPORTANT RESULTS/CONCEPTS

A matrix is a rectangular array of $m \times n$ numbers arranged in m rows and n columns.

$$A = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots a_{2n} \\ \\ a_{m1} & a_{m2} \ldots\ldots\ldots a_{mn} \end{bmatrix}_{m\times n} \quad \text{OR} \quad A = [a_{ij}]_{m\times n}, \text{ where } \mathbf{i} = 1, 2, \ldots, m \,;\ \mathbf{j} = 1, 2, \ldots, n.$$

* **Row Matrix**: A matrix which has one row is called row matrix. $A = [a_{ij}]_{1\times n}$

* **Column Matrix** : A matrix which has one column is called column matrix. $A = [a_{ij}]_{m\times 1}$.

* **Square Matrix**: A matrix in which number of rows are equal to number of columns, is called a square matrix $A = [a_{ij}]_{m\times m}$

* **Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all the elements, except the diagonal elements are zero. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, $\mathbf{i} \neq j$.

$a_{ij} \neq 0$, $\mathbf{i} = j$.

* **Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are same non-zero quantity.

$A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, $\mathbf{i} \neq j$.

$a_{ij} \neq \alpha$, $\mathbf{i} = j$.

* **Identity or Unit Matrix** : A square matrix in which all the non diagonal elements are zero and diagonal elements are unity is called identity or unit matrix.

*** Null Matrices** : A matrices in which all element are zero.
*** Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.
* **Transpose of matrix** : If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.\
* **Properties of Transpose** :
If A & B are matrices such that their sum & product are defined, then

(i). $\left(A^T\right)^T = A$ (ii). $\left(A+B\right)^T = A^T + B^T$ (iii). $\left(KA^T\right) = K.A^T$ where K is a scalar.
(iv). $\left(AB\right)^T = B^T A^T$ (v). $\left(ABC\right)^T = C^T B^T A^T$.

* **Symmetric Matrix** : A square matrix is said to be symmetric if $A = A^T$ i.e. If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = a_{ji}$ for all i, j. Also elements of the symmetric matrix are symmetric about the main diagonal
* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.
If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = -a_{ji}$ for all i, j.
***Singular matrix**:A square matrix 'A' of order 'n' is said to be singular, if $|A| = 0$.
*** Non -Singular matrix** : A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.

***Product of matrices:**

(i) If A & B are two matrices, then product AB is defined, if

Number of column of A = number of rows of B.

i.e. $A = [a_{ij}]_{m\times n}$, $B = [b_{jk}]_{n\times p}$ then AB = $AB = [C_{ik}]_{m\times p}$.

(ii) Product of matrices is not commutative. i.e. $AB \neq BA$.

(iii) Product of matrices is associative. i.e A(BC) = (AB)C

(iv) Product of matrices is distributive over addition.

***Adjoint of matrix** :

If $A = [a_{ij}]$ be a n-square matrix then transpose of a matrix $[A_{ij}]$,

where A_{ij} is the cofactor of A_{ij} element of matrix A, is called the adjoint of A.

Adjoint of A = Adj. A = $[A_{ij}]^T$.

A(Adj.A) = (Adj. A)A = | A| I.

***Inverse of a matrix** :Inverse of a square matrix A exists, if A is non-singular or square matrix

A is said to be invertible and $A^{-1} = \frac{1}{|A|}$ Adj.A

***System of Linear Equations** :

$a_1x + b_1y + c_1z = d_1$.
$a_2x + b_2y + c_2z = d_2$.
$a_3x + b_3y + c_3z = d_3$.

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow A\,X = B \;\; \Rightarrow X = A^{-1}B \quad ; \{\, |A| \neq 0\}.$$

***Criteria of Consistency**.

(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.

(ii) If $|A| = 0$ and (adj. A)B = 0, then the system of equations is consistent and has infinitely many solutions.

(iii) If $|A| = 0$ and (adj. A)B $\neq 0$, then the system of equations is inconsistent and has no solution.

*** Determinant** :

To every square matrix we can assign a number called determinant

If $A = [a_{11}]$, det. $A = |A| = a_{11}$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $|A| = a_{11}a_{22} - a_{21}a_{12}$.

*** Properties** :

(i) The determinant of the square matrix A is unchanged when its rows and columns are interchanged.

(ii) The determinant of a square matrix obtained by interchanging two rows(or two columns) is negative of given determinant.

(iii) If two rows or two columns of a determinant are identical, value of the determinant is zero.

(iv) If all the elements of a row or column of a square matrix A are multiplied by a non-zero number k, then determinant of the new matrix is k times the determinant of A.

If elements of any one column(or row) are expressed as sum of two elements each, then determinant can be written as sum of two determinants.

Any two or more rows(or column) can be added or subtracted proportionally.

If A & B are square matrices of same order, then $|AB| = |A|\,|B|$

ASSIGNMENTS

(i). Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. If a matrix has 5 elements, what are the possible orders it can have? **[CBSE 2011]**

2. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find A –2 B.

4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA.

LEVEL II

1. For the following matrices A and B, verify $(AB)^T = B^TA^T$,

   where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

2. Give example of matrices A & B such that AB = O, but BA ≠ O, where O is a zero matrix and

A, B are both non zero matrices.

3. If B is skew symmetric matrix, write whether the matrix (ABA^T) is Symmetric or skew symmetric.
4. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$

LEVEL III

1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 – 3A + 2I$

2. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where:

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

3. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \epsilon N$

(ii) Cofactors &Adjoint of a matrix

LEVEL I

1. Find the co-factor of a_{12} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
2. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

LEVEL II

Verify $A(adjA) = (adjA)\,A = |A|I$ if

1. $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A **CBSE 2011**
2. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A ?
3. For what value of k , the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible ?

LEVEL II

1. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 – 5A – 14I = 0$. Hence find A^{-1}

2. If A, B, C are three non zero square matrices of same order, find the condition on A such that $AB = AC \Rightarrow B = C$.

3. Find the number of all possible matrices A of order 3 × 3 with each entry 0 or 1 and for which A $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions.

LEVEL III

1 If A = $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

2x – 3y + 5z = 11, 3x + 2y – 4z = - 5, x + y – 2z = - 3

2. Using matrices, solve the following system of equations:

a. x + 2y - 3z = - 4
   2x + 3y + 2z = 2
   3x - 3y – 4z = 11 [**CBSE 2011]**

b. 4x + 3y + 2z = 60
   x + 2y + 3z = 45
   6x + 2y + 3z = 70 [**CBSE 2011]**

3. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

4. Using matrices, solve the following system of equations:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$

$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

5. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(iv)To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. Evaluate $\begin{vmatrix} \cos 15^{\circ} & \sin 15^{\circ} \\ \sin 75^{\circ} & \cos 75^{\circ} \end{vmatrix}$[**CBSE 2011]**

2. What is the value of $|3I|$, where I is identity matrix of order 3?

3. If A is non singular matrix of order 3 and $|A|$ = 3, then find $|2A|$

4. For what valve of a, $\begin{bmatrix} 2a & -1 \\ -8 & 3 \end{bmatrix}$ is a singular matrix?

LEVEL II

1. If A is a square matrix of order 3 such that $|adjA|$ = 64, find $|A|$
2. If A is a non singular matrix of order 3 and $|A|$ = 7, then find $|adjA|$

LEVEL III

1. If $A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$ and $|A|^3 = 125$, then find a.
2. A square matrix A, of order 3, has $|A| = 5$, find $|A.adjA|$

(v).Properties of Determinants

LEVEL I

1. Find positive valve of x if $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$

2. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

LEVEL II

1. Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

[**CBSE 2012]**

2. $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

3. $$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

4. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$ [**CBSE 2012]**

LEVEL III

1. Using properties of determinants, solve the following for x :

a. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ [**CBSE 2011]**

b. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ [**CBSE 2011]**

c. $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$ [**CBSE 2011]**

2. If a, b, c, are positive and unequal, show that the following determinant is negative:

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

3. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

4. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$ **[CBSE 2012]**

5. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

6. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

7. $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

8. If p, q, r are not in G.P and $\begin{vmatrix} 1 & \frac{q}{p} & \alpha + \frac{q}{p} \\ 1 & \frac{r}{q} & \alpha + \frac{r}{q} \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$, show that $p\alpha^2 + 2p\alpha + r = 0$.

9. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

Show that either a + b +c = 0 or a = b = c.

QUESTIONS FOR SELF EVALUTION

1. Using properties of determinants, prove that : $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

2. Using properties of determinants, prove that : $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

3. Using properties of determinants, prove that : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. .Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

5. Let $A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove by mathematical induction that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

6. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

7. Let $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

8. Solve the following system of equations : $x + 2y + z = 7$, $x + 3z = 11$, $2x - 3y = 1$.

9. Find the product AB, where $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

10. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

TOPIC 4
CONTINUITY AND DIFFRENTIABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Continuity& Differentiability	1.Limit of a function		
	2.Continuity	***	Ex 5.1 Q.No- 21, 26,30
	3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
	4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
	5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
	6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
	7. Mean Value Theorem	**	Ex 5.8 QNo- 3,4

SOME IMPORTANT RESULTS/CONCEPTS

* A function f is said to be continuous at x = a if Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = f(a)$

i.e. $\lim\limits_{h\to 0} f(a-h) = \lim\limits_{h\to 0} f(a-h) = f(a)$.

* A function is said to be differentiable at x = a if $Lf'(a) = Rf'(a)$ i.e

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

(i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$.

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) \quad = 0, \forall\, c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1$.

(v) $\frac{d}{dx}(e^x) = e^x$.

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x\log a}\ a > 0,\ a \neq 1,\ x$

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x}$, $x > 0$

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x,\ \forall\, x \in R$.

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x,\ \forall\, x \in R$.

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x,\ \forall\, x \in R$.

(xvi) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2},\ \forall\ x \in R$

(xix) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2},\ \forall\, x \in R$.

(xx) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$,.

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|\,x\,|) = \frac{x}{|\,x\,|},\ x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(viii) $\frac{d}{dx}(\log_a \lvert x \rvert) = \frac{1}{x \log a}$, $a > 0, a \neq 1, x \neq 0$ (ix) $\frac{d}{dx}(\log \lvert x \rvert) = \frac{1}{x}$, $x \neq 0$ (x) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R.$ (xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R.$ (xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R.$	(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$ (xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

2.Continuity

LEVEL-I

1.Examine the continuity of the function $f(x)=x^2 + 5$ at x=-1.

2. Examine the continuity of the function $f(x)=\frac{1}{x+3}$, $x \in R$.

3. Show that f(x)=4x is a continuous for all $x \in R$.

LEVEL-II

1. Give an example of a function which is continuous at x=1,but not differentiable at x=1.

2. For what value of k,the function $\begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at x=2.

3.Find the relationship between "a" and "b" so that the function 'f' defined by:

[CBSE 2011]

$f(x)=\begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

4. If $f(x)=\begin{cases} \frac{\sin 3x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$. Find whether f(x) is continuous at x=0.

LEVEL-III

1.For what value of k, the function $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x=0?

2. If function $f(x)=\frac{2x+3\sin x}{3x+2\sin x}$, for $x \neq 0$ is continuous at x=0, then Find f(0).

3.Let $f(x)\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2} \\ a & \text{if } x=\dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x>\dfrac{\pi}{2} \end{cases}$ = If f(x) be a continuous function at x=$\dfrac{\pi}{2}$, find a and b.

4.For what value of k,is the function f(x) = $\begin{cases} \dfrac{\sin x + x\cos x}{x} & \text{, when } x \neq 0 \\ k & \text{, when } x = 0 \end{cases}$ continuous at x= 0?

3.Differentiation

LEVEL-I

1. Discuss the differentiability of the function $f(x)=(x-1)^{2/3}$ at x=1.

2.Differentiate $y=\tan^{-1}\dfrac{2x}{1-x^2}$.

3. If $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\dfrac{dy}{dx}$.

LEVEL-II

1. Find $\dfrac{dy}{dx}$, $y = \cos(\log x)^2$.

2. Find $\dfrac{dy}{dx}$ of $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$

3.If $y=e^{ax}\sin bx$, then prove that $\dfrac{d^2y}{dx^2}-2a\dfrac{dy}{dx}+(a^2+b^2)y=0$.

4.Find $\dfrac{d^2y}{dx^2}$, if $y=\dfrac{3at}{1+t}$, $x=\dfrac{2at^2}{1+t}$.

LEVEL-III

1.Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

2.Find $\dfrac{dy}{dx}$ $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0<x<\dfrac{\pi}{2}$.

3. If $y=\sin^{-1}\left(\dfrac{a+b\cos x}{b+a\cos x}\right)$, show that $\dfrac{dy}{dx}=\dfrac{-\sqrt{b^2-a^2}}{b+a\cos x}$.

4.Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{1-x^2}\right)\right]=\frac{1}{1+x^4}$.

4.Logrithmic Differentiation

LEVEL-I

1.Differentiate $y=\log_7(\log x)$.

2. Differentiate , sin(log x),with respect to x.

3.Differentiate $y=\tan^{-1}(\log x)$

LEVEL-II

1. If y. $\sqrt{x^2+1}=\log[\sqrt{x^2+1}-x]$,show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.
2. Find $\frac{dy}{dx}$, $y = \cos(\log x)^2$.
3. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$ **[CBSE 2012]**

LEVEL-III

1.If $x^p.y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$

2. $y = (\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$

3. If $x = e^{x-y}$, Show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$ **[CBSE 2011]**

4. Find $\frac{dy}{dx}$ when $y = x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$ **[CBSE 2012]**

5 Parametric Differentiation

LEVEL-II

1.If y = tanx, prove that $\frac{d^2y}{dx^2}=2y\frac{dy}{dx}$

2. .If $x = a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$ find $\frac{d^2y}{dx^2}$at $\theta=\frac{\pi}{4}$.

3. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)=0$**[CBSE 2011]**

6. Second order derivatives

LEVEL-II

1. If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

2.If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$

3.If $(x-a)^2+(x-b)^2=c^2$ for some c>0.Prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant, independent

7. Mean Value Theorem

LEVEL-II

1.It is given that for the function $f(x)=x^3-6x^2+px+q$ on[1,3] , Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find the values p and q.

2. Verify Rolle's theorem for the function f(x) = sinx, in $[0,\pi]$.Find c, if verified

3.VeifyLagrange's mean Value Theorem $f(x)=\sqrt{x^2-4}$ in the interval [2,4]

Questions for self evaluation

1.For what value of k is the following function continuous at x = 2 ?

$$f(x)=\begin{cases}2x+1\,;\ x<2\\ k\,;\ x=2\\ 3x-1\,;\ x>2\end{cases}$$

2.If $f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11 & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$, continuous at x = 1, find the values of a and b.**[CBSE 2012 Comptt.]**

3. Discuss the continuity of $f(x)=|x-1|+|x-2|$ at x = 1 & x = 2.

4. If f(x), defined by the following is continuous at x = 0, find the values of a, b, c

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & ,x<0\\ c & ,x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & ,x>0\end{cases}$$

5.If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$.

6. If $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$.

7. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

8. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

9.If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

10. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

11.If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

12.If $x^p.y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

TOPIC 5
APPLICATIONS OF DERIVATIVES
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Application of Derivative.	1.Rate of change	*	Example 5 Ex 6.1 Q.No- 9,11
	2.Increasing & decreasing functions	***	Ex 6.2 Q.No- 6 Example 12,13
	3.Tangents & normals	**	Ex 6.3 Q.No- 5,8,13,15,23
	4.Approximations	*	Ex 6.4 QNo- 1,3
	5 Maxima & Minima	***	Ex 6.5Q.No- 8,22,23,25 Example 35,36,37,

SOME IMPORTANT RESULTS/CONCEPTS

** Whenever one quantity y varies with another quantity x, satisfying some rule y = f (x) , then $\frac{dy}{dx}$ (or $f'(x)$) represents the rate of change of y with respect to x and $\left[\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x = x_0$.

** Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \le f(x_2)$ for all $x_1, x_2 \in I$.

(ii) strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.

(iii) decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in I$.

(iv) strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$.

** (i) f is strictly increasing in (a, b) if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is strictly decreasing in (a, b) if $f'(x) < 0$ for each $x \in (a, b)$

(iii) A function will be increasing (decreasing) in **R** if it is so in every interval of **R**.

** Slope of the tangent to the curve y = f (x) at the point (x_0, y_0) is given by $\left[\frac{dy}{dx}\right]_{(x_0, y_0)} \left(= f'(x_0)\right)$.

** The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** Slope of the normal to the curve y = f (x) at (x_0, y_0) is $-\frac{1}{f'(x_0)}$.

** The equation of the normal at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

** If slope of the tangent line is zero, then $\tan\theta = 0$ and so $\theta = 0$ which means the tangent line is parallel to the x-axis. In this case, the equation of the tangent at the point (x0, y0) is given by $y = y_0$.

** If $\theta \to \frac{\pi}{2}$, then $\tan \theta \to \infty$, which means the tangent line is perpendicular to the x-axis, i.e., parallel to the y-axis. In this case, the equation of the tangent at (x_0, y_0) is given by $x = x_0$.

** Increment Δy in the function $y = f(x)$ corresponding to increment Δx in x is given by $\Delta y = \frac{dy}{dx}\Delta x$.

** Relative error in $y = \frac{\Delta y}{y}$.

** Percentage error in $y = \frac{\Delta y}{y} \times 100$.

** Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$.
The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) \leq f(x)$, for all $x \in I$.
The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

(c) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.
The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

* * Absolute maxima and minima

Let f be a function defined on the interval I and $c \in I$. Then

(a) f(c) is absolute minimum if $f(x) \geq f(c)$ for all $x \in I$.

(b) f(c) is absolute maximum if $f(x) \leq f(c)$ for all $x \in I$.

(c) $c \in I$ is called the critical point off if $f'(c) = 0$

(d) Absolute maximum or minimum value of a continuous function f on [a, b] occurs at a or b or at critical points off (i.e. at the points where $f'$is zero)

If $c_1, c_2, \dots, c_n$ are the critical points lying in [a , b], then

absolute maximum value of $f = \max\{f(a), f(c_1), f(c_2), \dots, f(c_n), f(b)\}$

and absolute minimum value of $f = \min\{f(a), f(c_1), f(c_2), \dots, f(c_n), f(b)\}$.

** Local maxima and minima

(a)A function f is said to have a local maxima or simply a maximum vaJue at x a if $f(a \pm h) \leq f(a)$ for sufficiently small h

(b)A function f is said to have a local minima or simply a minimum value at x = a if $f(a \pm h) \geq f(a)$.

** First derivative test : A function f has a maximum at a point x = a if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from + ve to –ve in the neighbourhood of ‘a’ (points taken from left to right).

However, f has a minimum at x = a, if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from –ve to +ve in the neighbourhood of ‘a’.

If $f'(a) = 0$ and f’(x) does not change sign, then f(x) has neither maximum nor minimum and the point ‘a’ is called point of inflation.

The points where $f'(x) = 0$ are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.

** Second derivative test

(i) a function has a maxima at x a if f ′(x) 0 and f ″ (a) <0
(ii) a function has a minima at x = a if f ′(x) = 0 and f ″(a) > 0.

ASSIGNMENTS

1.Rate of change

LEVEL -I

1. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.
2 .The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long ?
3. The radius of a circle is increasing at the rate of 0.7 cm/sec. what is the rate of increase of its circumference ?

LEVEL –II

1. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate?
2. A man 2 metre high walks at a uniform speed of 6km /h away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.
3. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm

LEVEL III

1. A particle moves along the curve $6y = x^3 + 2$., Find the points on the curve at which y-coordinate is changing 8 times as fast as the x-coordinate.
2. Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and altitude is 20 cm, Find the rate at which water level is dropping when it is 5 cm from top.
3. From a cylinder drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 ml/sec. If the radius of the drum is 10cm and height 50cm, find the rate at which the level of the petrol is changing when petrol level is 20 cm

2.Increasing & decreasing functions

LEVEL I

1. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$.
2. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0,1)
3. Find the intervals in which the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$ isincreasing or

decreasing.

LEVEL II

1. Indicate the interval in which the function f(x) = cos x, $0 \le x \le 2\pi$ is decreasing.

2.Show that the function $f(x) = \dfrac{\sin x}{x}$ is strictly decreasing on (0, $\pi/2$)

3. Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ increasing or decreasing.

LEVEL III

1. Find the interval of monotonocity of the function $f(x) = 2x^2 - \log x$, $x \neq 0$

2. Prove that the function $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in [0, $\pi/2$]

[CBSE 2011]

3.Tangents &Normals

LEVEL-I

1.Find the equations of the normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

2. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the x-coordinate of the point.

3. At what points on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x axis ?

LEVEL-II

1. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3)

2. For the curve $y = 2x^2 + 3x + 18$, find all the points at which the tangent passes through the origin.

3. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4= 0$

4. Show that the equation of tangent at (x_1 , y_1) to the parabola $yy_1=2a(x + x_1)$. **[CBSE 2012Comptt.]**

LEVEL- III

1 .Find the equation of the tangent line to the curve $y = \sqrt{5x-3}$ –2 which is parallel to the line $4x - 2y + 3 = 0$

2. Show that the curve $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0,0)

3. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.

4.Approximations

LEVEL-I

Q.1 Evaluate $\sqrt{25.3}$

Q.2 Use differentials to approximate the cube root of 66

Q.3 Evaluate $\sqrt{0.082}$

Q.4 Evaluate $\sqrt{49.5}$ **[CBSE 2012]**

LEVEL-II

1. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area

5 Maxima & Minima

LEVEL I

1. Find the maximum and minimum value of the function $f(x) = 3 - 2 \sin x$

2. Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value

3. Find two positive numbers whose sum is 24 and whose product is maximum

LEVEL II

1. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

2.A piece of wire 28(units) long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible.

3. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

LEVEL III

1 .Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one extremity of major axis.

2.An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.**[CBSE 2012 Comptt.]**

3.A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

[CBSE 2011]

Questions for self evaluation

1.Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

2. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

3. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = -2x^3 - 9x^2 - 12x + 1$

4. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = \sin x + \cos x ,\ 0 \le x \le 2\pi$

5. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$.

7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

8. Using differentials, find the approximate value of each of the following up to 3places of decimal :

(i) $(26)^{\frac{1}{3}}$ (ii) $(32.15)^{\frac{1}{5}}$

9. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

10. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

TOPIC 6
INDEFINITE & DEFINITE INTEGRALS
SCHEMATIC DIAGRAM

Topics	Concept	Degree of Importance	References Text book of NCERT, Vol. II 2007 Edition
Indefinite Integrals	(i) Integration by substitution	*	Exp 5&6 Page301,303
	(ii)) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
	(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	***	Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23
	(iv) Integration using Partial Fraction	***	Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
	(v) Integration by Parts	**	Exp 18,19&20 Page 325
	(vi)Some Special Integrals $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$	***	Exp 23 &24 Page 329
	(vii) Miscellaneous Questions	***	Solved Ex.41
Definite Integrals	(i) Definite Integrals based upon types of indefinite integrals	*	Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9
	(ii) Definite integrals as a limit of sum	**	Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
	(iii) Properties of definite Integrals	***	Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
	(iv) Integration of modulus function	**	Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

SOME IMPORTANT RESULTS/CONCEPTS

* $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
* $\int 1.dx = x + c$
* $\int \frac{1}{x^n} dx = -\frac{1}{x^n} + c$
* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$
* $\int \frac{1}{x} dx + c$
* $\int e^x dx = e^x + c$
* $\int a^x dx = \frac{a^x}{\log a} + c$
* $\int \sin x dx = -\cos x + c$
* $\int \sin x dx = -\cos x + c$
* $\int \cos x dx = \sin x + c$
* $\int \sec^2 x \, dx = \tan x + c$
* $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
* $\int \sec x . \tan x \, dx = \sec x + c$
* $\int \operatorname{cosec} x . \cot x \, dx = -\operatorname{cosec} x + c$
* $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + c$
* $\int \cot x \, dx = \log | \sin x | + C$
* $\int \sec x \, dx = \log | \sec x + \tan x | + C$

$= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$

* $\int \operatorname{cosec} x \, dx = \log | \operatorname{cosec} x - \cot x | + C$

$= -\log | \operatorname{cosec} x + \cot x | + C$

$= \log \left|\tan\frac{x}{2}\right| + C$

* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C, \text{ if } x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C, \text{ if } x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C, \text{ if } x > a$
* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + C$`
* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log | x + \sqrt{x^2 + a^2} | + C$
* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log | x + \sqrt{x^2 - a^2} | + C$
* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$
* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$
* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
* $\int \{f_1(x) \pm f_2(x) \pm ....f_n(x)\}dx$

$= \int f_1(x)dx \pm \int f_2(x)dx \pm ....\pm \int f_n(x)dx$

* $\int \lambda f(x) dx = \lambda \int f(x) dx + C$
* $\int u.v \, dx = u.\int v.dx - \int\left[\int v.dx\right]\frac{du}{dx}.dx$
* $\int_a^b f(x) \, dx = F(b) - F(a)$, where $F(x) = \int f(x) \, dx$

* **General Properties of Definite Integrals.**

* $\int_a^b f(x) \, dx = \int_a^b f(t) \, dx$
* $\int_a^b f(x) \, dx = -\int_a^b f(x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx$
* $\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx$
* $\int_{-a}^a f(x) \, dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$

$* \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C`$	$* \int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a - x) = f(x). \\ 0 & \text{if } f(2a - x) = -f(x) \end{cases}$

Assignments

(i) Integration by substitution

LEVEL I

1. $\int \frac{\sec^2(\log x)}{x} dx$
2. $\int \frac{e^{m\tan^{-1}x}}{1+x^2} dx$
3. $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} dx$

LEVEL II

1. $\int \frac{1}{\sqrt{x}+x} dx$
2. $\int \frac{1}{x\sqrt{x^6-1}} dx$
3. $\int \frac{1}{e^x-1} dx$

LEVEL III

1. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x} dx$
2. $\int \frac{\tan x}{\sec x + \cos x} dx$
3. $\int \frac{1}{\sin x.\cos^3 x} dx$

(ii) Application of trigonometric function in integrals

LEVEL I

1. $\int \sin^3 x.dx$
2. $\int \cos^2 3x.dx$
3. $\int \cos x.\cos 2x.\cos 3x.dx$

LEVEL II

1. $\int \sec^4 x.\tan x.dx$
2. $\int \frac{\sin 4x}{\sin x} dx$

LEVEL III

1. $\int \cos^5 x.dx$
2. $\int \sin^2 x.\cos^3 x.dx$

(iii) Integration using standard results

LEVEL I

1. $\int \frac{dx}{\sqrt{4x^2-9}}$
2. $\int \frac{1}{x^2+2x+10} dx$
3. $\int \frac{dx}{9x^2+12x+13}$

LEVEL II

1. $\int \frac{x}{x^4+x^2+1} dx$
2. $\int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$
3. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

LEVEL III

1. $\int \frac{2x}{\sqrt{1-x^2-x^4}}dx$ 2. $\int \frac{x^2+x+1}{x^2-x+1}dx$ 3. $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

4. $\int \sqrt{\frac{1-x}{1+x}}\,dx$ 5. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}$ **[CBSE 2011]**

(iv) Integration using Partial Fraction

LEVEL I

1. $\int \frac{2x+1}{(x+1)(x-1)}dx$ 2. $\int \frac{x^2}{(x-1)(x-2)(x-3)}dx$ 3. $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

LEVEL II

1. $\int \frac{x^2+2x+8}{(x-1)(x-2)}dx$ 2. $\int \frac{x^2+x+1}{x^2(x+2)}dx$ 3. $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

LEVEL III

1. $\int \frac{8}{(x+2)(x^2+4)}dx$ 2. $\int \frac{dx}{\sin x+\sin 2x}$ 3. $\int \frac{1}{1+x^3}dx$

(v) Integration by Parts

LEVEL I

1. $\int x.\sec^2 x.dx$ 2. $\int \log x.dx$ 3. $\int e^x(\tan x+\log\sec x)dx$

LEVEL II

1. $\int \sin^{-1} x.dx$ 2. $\int x^2.\sin^{-1} x.dx$ 3. $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

4. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).dx$ 5. $\int \sec^3 x.dx$

LEVEL III

1. $\int \cos(\log x)dx$ 2. $\int \frac{e^x(1+x)}{(2+x)^2 dx}$ 3. $\int \frac{\log x}{(1+\log x)^2}dx$

4. $\int \frac{2+\sin x}{1+\cos 2x}e^x.dx$ 5. $\int e^{2x}.\cos 3x.dx$

(vi) Some Special Integrals

LEVEL I

1. $\int \sqrt{4+x^2}.dx$ 2. $\int \sqrt{1-4x^2}.dx$

LEVEL II

1. $\int \sqrt{x^2+4x+6}.dx$ 2. $\int \sqrt{1-4x-x^2}.dx$

LEVEL III

1. $\int (x+1)\sqrt{1-x-x^2}.dx$ 2. $\int (x-5)\sqrt{x^2+x}\ dx$

(vii) Miscellaneous Questions

LEVEL II

1. $\int \frac{1}{2-3\cos 2x}dx$ 2. $\int \frac{1}{3+\sin 2x}dx$ 3. $\int \frac{dx}{4\sin^2 x+5\cos^2 x}$

4. $\int \frac{dx}{1+3\sin^2 x+8\cos^2 x}$ 5. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ 6. $\int \frac{\sec x}{5\sec x+4\tan x}dx$

LEVEL III

1. $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$ 2. $\int \frac{dx}{1-\tan x}$ 3. $\int \frac{x^4}{x^4-1}dx$

4. $\int \frac{x^2+1}{x^4+x^2+1}dx$ 5. $\int \frac{x^2-1}{x^4+1}dx$ 6. $\int \sqrt{\tan x}.dx$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\int_0^1 \frac{2x+3}{5x^2+1}$ 2. $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$ 3. $\int_0^2 x\sqrt{x+2}\,dx$

LEVEL II

1. $\int_1^2 \frac{5x^2}{x^2+4x+3}dx$ 2. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}\ dx$

(ii) Definite integrals as a limit of sum

LEVEL I

1. Evaluate $\int_0^2 (x+2)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^4 (1+x)\,dx$ definite integral as the limit of a sum.

LEVEL II

1. Evaluate $\int_1^2 (3x^2 - 1)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2 + 1)\,dx$ as the limit of a sum.

LEVEL III

1. Evaluate $\int_1^2 (x^2 + x + 2)dx$ as the limit of a sum.

2. Evaluate $\int_2^4 \left(e^{2x} + x^2\right)dx$ as the limit of a sum.

(iii) Properties of definite Integrals

LEVEL I

1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$ 2. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$ 3. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$

LEVEL II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$ 2. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx$ 3. $\int_0^{\pi} \frac{x \tan x}{\sec x.\operatorname{cosec}}dx$

4. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **[CBSE 2011]**

LEVEL III

1. $\int_0^{\pi} \frac{x+\sin x}{1+\cos x}dx$ **[CBSE 2011]** 2. $\int_0^{\pi/2} \log \sin x\ dx$ 3. $\int_0^{\pi/4} \log(1+\tan x)dx$

[CBSE 2011]

(iv) Integration of modulus function

LEVEL III

1. $\int_2^5 (|x-2|+|x-3|+|x-4|)dx$ 2. $\int_{-1}^2 |x^3 - x|dx$ 3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|]dx$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2-3x-18}$ 2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x.dx$

4. Evaluate $\int \frac{dx}{3+2\sin x+\cos x}$

5. Evaluate $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$

6. Evaluate $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$

8. Evaluate $\int_{-1}^{3/2} |x \sin \pi x|\, dx$

9. Evaluate $\int_0^{\pi/2} \log \sin x\, dx$

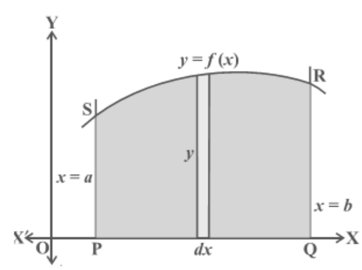
10. Evaluate $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

TOPIC 7
APPLICATIONS OF INTEGRATION
SCHEMATIC DIAGRAM

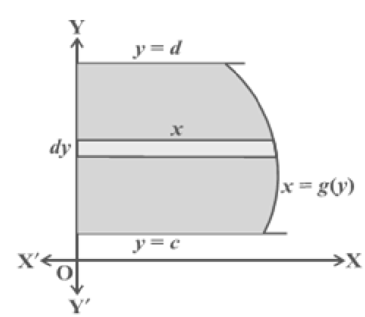
Topic	Concepts	Degree of Importance	Reference NCERT Text BookEdition 2007
Applications of Integration	(i)Area under ***Simple Curves***	*	Ex.8.1 Q.1,2,5
	(ii) Area of the region enclosed between ***Parabola and line***	***	Ex. 8.1 Q 10,11 Misc.Ex.Q 7
	(iii) Area of the region enclosed between ***Ellipse and line***	***	Example 8, page 369 Misc.Ex. 8
	(iv) Area of the region enclosed between***Circle and line***	***	Ex. 8.1 Q 6
	(v) Area of the region enclosed between ***Circle and parabola***	***	Ex 8.2 Q1, Misc.Ex.Q 15
	(vi) Area of the region enclosed between ***Two Circles***	***	Example 10, page370 Ex 8.2 Q2
	(vii) Area of the region enclosed between ***Two parabolas***	***	Example 6, page368
	(viii) Area of triangle ***when vertices are given***	***	Example 9, page370 Ex 8.2 Q4
	(ix) Area of triangle ***when sides are given***	***	Ex 8.2 Q5 ,Misc.Ex. Q 14
	(x) ***Miscellaneous Questions***	***	Example 10, page374 Misc.Ex.Q 4, 12

SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP $= \int_a^b dA = \int_a^b y\,dx = \int_a^b f(x)\,dx$.



** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by $A = \int_c^d x\,dy = \int_c^d g(y)\,dy$



ASSIGNMENTS

(i) Area under ***Simple Curves***

LEVEL I

1. Sketch the region of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and find its area, using integration,

.

2. Sketch the region $\{(x, y) : 4x^2 + 9y^2 = 36\}$ and find its area, using integration.

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II

1. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.
2. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

LEVEL III

1. Find the area of the region : $\{(x, y): y \le x^2 + 1,\ y \le x + 1, 0 \le x \le 2\}$

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II

1. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ and the straight line $\frac{x}{4}+\frac{y}{5}=1$.

(iv) Area of the region enclosed between ***Circle and line***

LEVEL II

1. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

LEVEL III

1. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$

(v) Area of the region enclosed between ***Circle and parabola***

LEVEL III

1. Draw the rough sketch of the region $\{(x, y): x^2 \le 6y, x^2 + y^2 \le 16\}$ an find the area enclosed by the region using the method of integration.
2. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III

1. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II

1. Draw the rough sketch and find the area of the region bounded by two parabolas

$4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

(viii) Area of triangle ***when vertices are given***

LEVEL III

1. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).
2. Using integration compute the area of the region bounded by the triangle whose vertices are (–1 , 1), (0 , 5), and (3 , 2).

(ix) Area of triangle ***when sides are given***

LEVEL III

1. Using integration find the area of the region bounded by the triangle whose sides are
$y = 2x + 1$, $y = 3x + 1$, $x = 4$.

2. Using integration compute the area of the region bounded by the linesx + 2y = 2, y – x =1, and 2x + y = 7.

(x) Miscellaneous Questions

LEVEL III

1. Find the area of the region bounded by the curves $y = |x-1|$ and $y = -|x-1| + 1$.
2. Find the area bounded by the curve $y = x$ and $y = x^3$.
3. Draw a rough sketch of the curve y = sinx and y = cosx as x varies from x = 0 to $x = \frac{\pi}{2}$ and find the area of the region enclosed by them and x-axis
4. Sketch the graph of $y = |x+1|$ .Evaluate $\int_{-3}^{1} |x+1|\,dx$ .What does this value represent on the graph.
5. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.
6. Sketch the graph of $y = |x+3|$ and evaluate the area under the curve $y = |x+3|$ above x-axis and between $x = -6$ to $x = 0$. **[CBSE 2011]**

Questions for self evaluation

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
2. Find the area bounded by the parabola $y = x^2$ and $y = |x|$.
3. Find the area of the region : $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$
4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.
5. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1, \le x + y\}$
6. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.
7. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

8. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).

9. Using integration compute the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$, and $x - 3y + 5 = 0$.

10. Sketch the graph of : $f(x) = \begin{cases} |x-2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$.

Evaluate $\int_0^4 f(x)dx$. What does the value of this integral represent on the graph ?

TOPIC 8
DIFFERENTIAL EQUATIONS
SCHEMATIC DIAGRAM

		(ii).General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		(iii).Formation of differential equation whose general solution is given	*	Q. 7,8,10 pg 391
		(iv).Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		(vi).Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406
		(vii)Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414

SOME IMPORTANT RESULTS/CONCEPTS

** Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.

** Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives are of the given differential equation is called the degree of the differential equatin

** Homogeneous Differential Equation : $\frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)}$, where $f_1(x, y)$ & $f_2(x, y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

i. $\frac{dy}{dx} + py = q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx} = \int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

ii. $\frac{dx}{dy} + px = q$, where p & q be the function of y or constant.

Solution of the equation is : $x.e^{\int p\,dy} = \int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

ASSIGNMENTS

1. *Order and degree of a differential equation*

LEVEL I

1. Write the order and degree of the following differential equations

   (i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

2. *General and particular solutions of a differential equation*

LEVEL I

1. Show that $y = e^{-x} + ax + b$ is the solution of $e^x \frac{d^2y}{dx^2} = 1$

3. *Formation of differential equation*

LEVEL II

1. Obtain the differential equation by eliminating a and b from the equation y = e^x(acosx + bsinx)

LEVEL III

1. Find the differential equation of the family of circles $(x - a)^2 - (y - b)^2 = r^2$

2. Obtain the differential equation representing the family of parabola having vertex at the origin and axis along the positive direction of x-axis

4. *Solution of differential equation by the method of separation of variables*

LEVEL II

1. Solve $\frac{dy}{dx} = 1 + x + y + xy$ 2. Solve $\frac{dy}{dx} = e^{-y}\cos x$ given that y(0)=0.

3. Solve $\left(1 + x^2\right)\frac{dy}{dx} - x = \tan^{-1} x$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II

1. Solve $(x^2 + xy)dy = (x^2 + y^2)dx$

LEVEL III

Show that the given differential equation is homogenous and solve it.

1. $(x - y)\frac{dy}{dx} = x + 2y$ 2. $ydx + x\log(\frac{y}{x})dy - 2xdy = 0$

3.Solve $xdy - ydx = \sqrt{x^2 - y^2}\,dx$ 4.Solve $x^2 y dx - (x^3 + y^3)dy = 0$

5.Solve $xdy - ydx = \sqrt{(x^2 + y^2)}\,dx$ **CBSE2011** 6.Solve $\left(y + 3x^2\right)\frac{dx}{dy} = x$

7. Solve $x\,dy + (y - x^3)dx = 0$ **CBSE2011** 8.Solve $x\,dy + (y + 2x^2)dx = 0$

6. *Linear Differential Equations*

LEVEL I

1.Find the integrating factor of the differential $x\frac{dy}{dx} - y = 2x^2$

LEVEL II

1.Solve $\frac{dy}{dx} + 2y\tan x = \sin x$ 2. Solve $(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

3. Solve $x\frac{dy}{dx} + y = x\log x$

LEVEL III

1. Solve $\frac{dy}{dx} = \cos(x+y)$ 2.Solve $ye^y dx = (y^3 + 2xe^y)dy$

3. Solve $x^2\frac{dy}{dx} = y(x+y)$ 4. Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = -\frac{1}{(x^2+1)^3}$

5. Solve the differential equation $\left(x + 2y^2\right)\frac{dy}{dx} = y$;given that when x=2,y=1

Questions for self evaluation

1. Write the order and degree of the differential equation $\left(\frac{d^3y}{dy^3}\right)^2 + \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$

2. Form the differential equation representing the family of ellipses having foci on x –axis and centre at origin .

3. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that y = 0 when x = 0.

4. Solve the differential equation :xdy – y dx = $\sqrt{x^2 + y^2}\,dx$

5. Solve the differential equation : $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$.

6. Solve the differential equation : $x^2 dy + (y^2 + xy)\,dx = 0$, y(1) = 1

7. Show that the differential equation $2y.e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous andfind its particular solution given that y(0) = 1 .

8. Find the particular solution of differential equation

$$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x,\ \text{given that}\ y\left(\frac{\pi}{2}\right) = 0\ .$$

TOPIC 9
VECTOR ALGEBRA
SCHEMATIC DIAGRAM

Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
Vector algebra	(i)Vector and scalars	*	Q2 pg428
	(ii)Direction ratio and direction cosines	*	Q 12,13 pg 440
	(iii)Unit vector	* *	Ex 6,8 Pg 436
	(iv)Position vector of a point and collinear vectors	* *	Q 15 Pg 440 , Q 11Pg440 , Q 16 Pg448
	(v)Dot product of two vectors	**	Q6 ,13 Pg445
	(vi)Projection of a vector	* * *	Ex 16 Pg 445
	(vii)Cross product of two vectors	* *	Q 12 Pg458
	(viii)Area of a triangle	*	Q 9 Pg 454
	(ix)Area of a parallelogram	*	Q 10 Pg 455

SOME IMPORTANT RESULTS/CONCEPTS

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

*Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

* Scalar Product (dot product) between two vectors : $\vec{a} \, . \, \vec{b} = |\vec{a}||\vec{b}| \cos\theta$; θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a} \, . \, \vec{b}}{|\vec{a}||\vec{b}|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a} \, . \, \vec{b} = a_1 a_2 + b_1 b_2 + c_1 c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}\cdot\vec{b} = 0$

* $\vec{a}\cdot\vec{a} = \left|\vec{a}\right|^2$

* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|}$

* Vector product between two vectors:

  $\vec{a}\times\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\ \hat{n}$; $\hat{n}$ is the normal unit vector which is perpendicular to both $\vec{a}$ & $\vec{b}$

* $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}\times\vec{b} = 0$

* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= \left|\vec{a}\times\vec{b}\right|$

* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

ASSIGNMENTS

(i) Vector and scalars, Direction ratio and direction cosines&Unit vector

LEVEL I

1. If $\vec{a} = \hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 3\hat{k}$ find a unit vector parallel to $\vec{a} + \vec{b}$
2. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
3. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$; $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$ find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$
4. Find a unit vector in the direction of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ **[CBSE 2011]**
5. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$, whose magnitude is 7

LEVEL II

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors ($\vec{a} + \vec{b}$),($\vec{a} - \vec{b}$) where

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

2. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

LEVEL – III

1. If a line make α,β, γ with the X - axis , Y– axis and Z – axis respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
2. For what value of p, is $(\hat{i} + \hat{j} + \hat{k})\,p$ a unit vector?
3. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis
4. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

(ii)Position vector of a point and collinear vectors

LEVEL – I

1. Find the position vector of the midpoint of the line segment joining the points A($5\hat{i} + 3\hat{j}$) and B($3\hat{i} - \hat{j}$).

2. In a triangle ABC, the sides AB and BC are represents by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

3. Show that the points (1,0), (6,0) ,(0,0) are collinear.

LEVEL – II

1.Write the position vector of a point R which divides the line joining the points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2 : 1 externally.

2.Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ

(iii) Dot product of two vectors

LEVEL – I

1.Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$.

2.If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

3.Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$ **[CBSE 2011]**

LEVEL – II

1. The dot products of a vector with the vectors $\hat{i} - 3\hat{j}$, $\hat{i} - 2\hat{j}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vectors.

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$.

3. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ.

LEVEL – III

1. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$.

2. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

3. For what values of λ, vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} = \lambda\hat{i} - 4\hat{j} + 8\hat{k}$ are
   (i) Orthogonal (ii) Parallel `

4..Find$|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})=15$.

5. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \mu\hat{k}$, find μ, such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

6. Show that the vector $2\hat{i} - \hat{j} + \hat{k}$, $-3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form sides of a right angled triangle.

7.Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

8. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

9. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each of them being perpendicular

to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

(iv) Projection of a vector

LEVEL – I

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\cdot\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ **[CBSE 2011]**

3.Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

LEVEL – II

1.Three vertices of a triangle are A(0, -1, -2), B(3,1,4) and C(5,7,1). Show that it is a right angled triangle. Also find the other two angles.

2.Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

3.If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non - zero and non – coplanar vectors, prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are also coplanar

LEVEL – III

1.If a unit vector $\vec{a}$ makes angles $\pi/4$, with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$,and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

4.Show that the points A, B, C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

5. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that

$$\text{(i)}\ \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}| \quad \text{(ii)}\ \tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$$

(vii)Cross product of two vectors

LEVEL – I

1. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$. Find $|\vec{a} \times \vec{b}|$

2. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

3. Find $|\vec{x}|$, if $\vec{p}$ is a unit vector and, $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$.

4. Find p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

LEVEL – II

1. Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

2. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.

4. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{a} \times \vec{b})$.

LEVEL – III

1. Find the value of the following: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{i}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$

2. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $(\vec{a} - \vec{d})$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

5. Express $2\hat{i}-\hat{j}+3\hat{k}$ as the sum of a vector parellal and perpendicular to $2\hat{i}+4\hat{j}-2\hat{k}$.

(viii)Area of a triangle & Area of a parallelogram

LEVEL – I

1.Find the area of Parallelogram whose adjacent sides are represented by the vectors

$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

2.If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a Parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

3. Find the area of triangle having the points A(1,1,1) , B(1,2,3) and C(2,3,1) as its vertices.

LEVEL – II

1.Show that the area of the Parallelogram having diagonals ($3\hat{i} + \hat{j} - 2\hat{k}$) and ($\hat{i} - 3\hat{j} + 4\hat{k}$) is $5\sqrt{3}$ Sq units.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a ΔABC, show that the area of the ΔABC is

$$\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|.$$

3.Using Vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5)

[CBSE 2011]

Questions for self evaluation

1.The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

3. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. Dot product of a vector with $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$, and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, 8 respectively. Find the vector.

5. Find the components of a vector which is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

TOPIC 10
THREE DIMENSIONAL GEOMETRY
SCHEMATIC DIAGRAM

Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
Three Dimensional Geometry	(i) Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
	(ii)Cartesian and Vector equation of a line in space & conversion of one into another form	**	Ex No 8 Pg -470 Q N. 6, 7, - Pg 477 QN 9 – Pg 478
	(iii)Co-planer and skew lines	*	Ex No 29 Pg -496
	(iv) Shortest distance between two lines	***	Ex No 12 Pg -476 Q N. 16, 17 - Pg 478
	(v) Cartesian and Vector equation of a plane in space & conversion of one into another form	**	Ex No 17 Pg -482 Ex No 18 Pg – 484 Ex No 19 Pg – 485 Ex No 27 Pg – 495 Q N. 19, 20 - Pg 499
	(vi) Angle Between (iv) Two lines (v) Two planes (vi) Line & plane	 * * **	Ex No 9 Pg -472 Q N. 11 - Pg 478 Ex No 26 Pg – 494 Q N. 12 - Pg 494 Ex No 25 Pg - 492
	(vii) Distance of a point from a plane	**	Q No 18 Pg -499 Q No 14 Pg – 494
	(viii)Distance measures parallel to plane and parallel to line	**	
	(ix)Equation of a plane through the intersection of two planes	***	Q No 10 Pg -493
	(x) Foot of perpendicular and image with respect to a line and plane	**	Ex. N 16 Pg 481

SOME IMPORTANT RESULTS/CONCEPTS

** Direction cosines and direction ratios :

If a line makes angles α, β and γ with x, y and z axes respectively the $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines denoted by l, m, n respectively and $l^2 + m^2 + n^2 = 1$

Any three numbers proportional to direction cosines are direction ratios denoted by a, b, c

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c} \qquad l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}},$$

* Direction ratios of a line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ may be taken as $x_2-x_1,\ y_2-y_1,\ z_2-z_1$
* Angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 is given by

$$\cos\theta = l_1l_2+m_1m_2+n_1n_2=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\left(a_1{}^2+b_1{}^2+c_1{}^2\right)\left(a_2{}^2+b_2{}^2+c_2{}^2\right)}}$$

* For parallel lines $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and

for perpendicular lines $a_1a_2+b_1b_2+c_1c_2=0$ or $l_1l_2+m_1m_2+n_1n_2=0$

** STRAIGHTLINE:

* Equation of line passing through a point (x_1, y_1, z_1) with direction cosines a, b, c: $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$
* Equation of line passing through a point (x_1, y_1, z_1) and parallel to the line: $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

* Equation of line passing through two point (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$
* Equation of line (Vector form)

Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

* Equation of line passing through two points $\vec{a}$ & $\vec{b}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$
* Shortest distance between two skew lines: if lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$

then Shortest distance $=\frac{\left(\vec{a_2}-\vec{a_1}\right)\cdot\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}$; $\vec{b_1}\times\vec{b_2}\neq 0$

$$\frac{\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b_1}\right|}{\left|\vec{b_1}\right|} \quad ;\vec{b_1}\times\vec{b_2}=0$$

** PLANE:

* Equation of plane is $ax + by + cz + d = 0$ where a, b & c are direction ratios of normal to the plane
* Equation of plane passing through a point (x_1, y_1, z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
* Equation of plane in intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where a, b, c are intercepts on the axes
* Equation of plane in normal form $lx+my+nz=p$ where l, m, n are direction cosines of normal to the plane p is length of perpendicular form origin to the plane

* Equation of plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and $(x_3, y_3, z\ \ )$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

* Equation of plane passing through two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and perpendicular to the plane

$a_1x + b_1y + c_1z + d_1 = 0$ or parralal to the line $\frac{x - \alpha_1}{a_1} = \frac{y - \beta_1}{b_1} = \frac{z - \gamma_1}{c_1}$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$

* Equation of plane passing through the point (x_1, y_1, z_1) and perpendicular to the planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z\ \ + d_2 = 0$ or parralal to the lines $\frac{x - \alpha_1}{a_1} = \frac{y - \beta_1}{b_1} = \frac{z - \gamma_1}{c_1}$

and $\frac{x - \alpha_2}{a_2} = \frac{y - \beta_2}{b_2} = \frac{z - \gamma_2}{c_2}$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane contaning the line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and passing through the point (x_2, y_2, z_2) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

* Condition for coplaner lines : $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ are coplaner if

$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ and equation of common plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane passing through the intersection of two planes $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$ is $(a_1x + b_1y + c_1z) + \lambda(a_2x + b_2y + c_2z) = 0$

* Perpendicular distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

* Distance between two parallel planes $ax + by + cz + d_1 = 0$, $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

ASSIGNMENTS

(i)Direction Ratios and Direction Cosines

LEVEL-I

1. Write the direction-cosines of the line joining the points (1,0,0) and (0,1,1) **[CBSE 2011]**

2.Find the direction cosines of the line passing through the following points (-2,4,-5), (1,2,3).

3.Write the direction cosines of a line equally inclined to the three coordinate axes

LEVEL-II

1**.**Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$.

2.Write the direction ratios of a line parallel to the line $\frac{5-x}{3} = \frac{y+7}{-2} = \frac{z+2}{6}$.

3. If the equation of a line AB $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+6}{3}$ Find the direction cosine.

4. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axis.

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL-I

1.Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$. **[CBSE 2011]**

2. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3).

3.Express the equation of the plane $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ in the Cartesian form.

4.Express the equation of the plane $\vec{r} \cdot (2\hat{\imath} - 3\hat{\jmath} + \hat{k}) + 4 = 0$ in the Cartesian form.

(iii) Co-planer and skew lines

LEVEL-II

1.Find whether the lines $\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ intersect or not. If intersecting , find their point of intersection.

2.Show that the four points (0,-1,-1) , (4,5,1) , (3,9,4) and (-4,4,4,) are coplanar. Also, find the equation of the plane containing them.

3.Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

LEVEL-III

1. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

2. The points A(4,5,10) , B(2,3,4) and C(1,2,-1) are three vertices of a parallelogram ABCD. Find

the vector equation of the sides AB and BC and also find the coordinates

3.Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1,1,1).
4. Show that The four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

(iv) Shortest distance between two lines

LEVEL-II

1. Find the shortest distance between the lines l_1 and l_2 given by the following:

(a) $l_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ $\quad$ $l_2 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

(b) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$
$\vec{r} = (4\hat{\imath} + 2\mu)\hat{\imath} + (5 + 3\mu)\hat{\jmath} + (6 + \mu)\hat{k}.$

2. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.
3.. Find the shortest distance between the lines
$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$
4.Find the shortest distance between the lines
$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{\jmath} + (3 - t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} + (2s + 1)\hat{k}$ **[CBSE 2011]**

5. Find the distance between the parallel planes x + y – z = -4 and 2x + 2y – 2z + 10 = 0.

6. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-}{2} = \frac{z+1}{4}$ and passing through (3,0,-4). Also, find the distance between these two lines.

(v) Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1.Find the equation of a plane passing through the origin and perpendicular to x-axis
2.Find the equation of plane with intercepts 2, 3, 4 on the x ,y, z –axis respectively.
3.Find the direction cosines of the unit vector perpendicular to the plane

$\vec{r} \cdot (6\,\hat{i} - 3\,\hat{j} - 2\,\hat{k}) + 1 = 0$ passing through the origin.

4.Find the Cartesian equation of the following planes:

(a) $r \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ $\qquad$ (b) $r \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

LEVEL II

1. Find the vector and cartesian equations of the plane which passes through the point (5, 2, – 4) and perpendicular to the line with direction ratios 2, 3, – 1.
2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector 3 iˆ + 5 ˆj - 6 kˆ.
3.Find the vector and cartesian equations of the planes that passes through the point (1, 0, – 2)and the normal to the plane is iˆ + ˆj - kˆ.

(vi) Angle Between(i)Two lines (ii)Two planes (iii)Line & plane

LEVEL-I

1. Find the angle between the lines whose direction ratios are (1, 1, 2) and ($\sqrt{3}$–1 , –$\sqrt{3}$ –1 , 4).

2.Find the angle between line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0.

3.Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane
3x – y – 2z = 7.

4.Find the angle between the planes whose vector equations are
r· (2 iˆ + 2 ˆj - 3 kˆ) = 5 and r · (3 iˆ - 3 ˆj + 5 kˆ) = 3

5.Find the angle between the line$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$and the plane 10 x + 2y – 11 z = 3.

LEVEL-II

1.Find the value of p, such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2p}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.
2. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

(vii) Distance of a point from a plane

LEVELI

1.Write the distance of plane 2x– y + 2z + 1 = 0 from the origins.
2.Find the point through which the line 2x = 3y = 4z passes.
3. Find the distance of a point $(2, 5, -3)$from the plane $r \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$
4. Find the distance of the following plane from origin: 2x – y + 2z + 1 = 0

5.Find the distance of the point (a,b,c) from x-axis

LEVELII

1..Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

2.Find the distance of the point (3,4,5) from the plane x + y + z =2 measured parallel to the line 2x = y = z.

3. Find the distance between the point P(6, 5, 9) and the plane determinedby the points A (3, – 1, 2), B (5, 2, 4) and C(– 1, – 1, 6).

4.Find the distance of the point $(-1, -5, -10)$ from the point of intersection of theline

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right) \text{ and the plane } \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

[**CBSE2011**]

LEVEL III

1.Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane 2x – y + z + 3 = 0. Find also, the image of the point in the plane.

2.Find the distance of the point P(6,5,9) from the plane determined by the points A(3,-1,2) , B(5,2,4) and C(-1,-1,6).

3.Find the equation of the plane containing the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \mu(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$.Find the distance of this plane from origin and also from the point (1,1,1).

(viii) Equation of a plane through the intersection of two planes

LEVELII

1.Find the equation of plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the perpendicular distance of the plane from the origin.

2.Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z +8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

3.Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of the planes 2x + 3y -2 z = 5 and x + 2y – 3z = 8.

LEVEL-III

1.Find the equation of the plane passing through the point (1,1,1) and containing the line $\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} - 5\hat{k})$. Also, show that the plane contains the line $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

2.Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

3.Find the Cartesian equation of the plane passing through the points A(0,0,0) and B(3,-1,2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

4. Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line

$$\frac{x+5}{1} = \frac{+3}{4} = \frac{z-6}{-9}.$$

(ix)Foot of perpendicular and image with respect to a line and plane

LEVEL II

1. Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A(1,2,3) , B(2,2,1) and C(-1,3,6).

2. Find the foot of the perpendicular from P(1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point (1,2,3).

3.Prove that the image of the point (3,-2,1) in the plane 3x – y + 4z =2 lies on the plane, x + y + z + 4 = 0.

LEVEL-III

1.Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).

2. Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.

3. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane

4. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane 2x – y+z+1=0. Find also, the image of the point in the plane.

Questions for self evaluation

1. Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

2. Find the vector equation of a line joining the points with position vectors $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$, and $2\hat{i} + \hat{j} + 2\hat{k}$. Also find the cartesian equivalent of this equation.

3. Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).

4. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

5.Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.

6. Show that the four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

7. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane.

8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

9. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

TOPIC 11
LINEAR PROGRAMMING
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Linear Programming	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
	(ii) Graphical method of solving LPP (bounded and unbounded solutions)	**	**Article 12.2.2** Solved Ex. 1 to 5 Q. Nos 5 to 8 Ex.12.1
	(iii) Diet Problem	***	Q. Nos 1, 2 and 9 Ex. 12.2 Solved Ex. 9 Q. Nos 2 and 3 Misc. Ex. Solved Ex. 8 Q. Nos 3,4,5,6,7 of Ex.
	(iv) Manufacturing Problem	***	12.2 Solved Ex.10 Q. Nos 4 & 10 Misc. Ex.
	(v) Allocation Problem	**	Solved Example 7Q. No 10 Ex.12.2, Q. No 5 & 8 Misc. Ex.
	(vi) Transportation Problem	*	Solved Ex.11 Q. Nos 6 & 7 Misc. Ex.
	(vii) Miscellaneous Problems	**	Q. No 8 Ex. 12.2

SOME IMPORTANT RESULTS/CONCEPTS

** Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = $ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is **bounded**, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is **unbounded**, we have:
4. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

ASSIGNMENTS

(i) LPP and its Mathematical Formulation

LEVEL I

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food ‘I’ contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food ‘II’ contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food ‘I’ and Rs 70 per kg to purchase Food ‘II’. Formulate this problem as a linear programming problem.

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

LEVEL I

Solve the following Linear Programming Problems graphically:

1.Minimise $Z = -3x + 4y$ subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$.

2.Maximise $Z = 5x + 3y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.

3.Minimise $Z = 3x + 5y$ suchthat $x + 3y \ge 3$, $x + y \ge 2$, $x, y \ge 0$.

(iii) Diet Problem

LEVEL II

1.A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost? Also find the least cost.

2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost ?

(iv) Manufacturing Problem

LEVEL II

1.A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

2. A company sells two different produces A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs. 20 and that on B is Rs. 15. How many units of A and B should be produced to maximize the profit? Solve it graphically

LEVEL III

1.A manufacture makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below:

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

(v) Allocation Problem

LEVEL II

1. Ramesh wants to invest at most Rs. 70,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10,000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8 % per annum and the rate of interest on bond B is 10 % per annum , how much money should he invest to earn maximum yearly income ? Find also his maximum yearly income.

2. An oil company requires 12,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

LEVEL III

1. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

(vi) Transportation Problem

LEVEL III

1. A medicine company has factories at two places A and B . From these places, supply is to be made to each of its three agencies P, Q and R. The monthly requirement of these agencies are respectively 40, 40 and 50 packets of the medicines, While the production capacity of the factories at A and B are 60 and 70 packets are respectively. The transportation cost per packet from these factories to the agencies are given:

Transportation cost per packet (in Rs.)		
From / To	**A**	**B**
P	**5**	**4**
Q	**4**	**2**
R	**3**	**5**

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

Questions for self evaluation

1. Solve the following linear programming problem graphically : Maximize $z = x - 7y + 190$ subject to the constraints $x + y \leq 8$, $x \leq 5$, $y \leq 5$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following linear programming problem graphically : Maximize $z = 3x + 5y$ subject to the constraints $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.

3. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains, 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kilogram and rice costs Rs. 4 per kilogram.

4. A shopkeeper deals only in two items — tables and chairs. He has Rs. 6,000 to invest and a space to store at most 20 pieces. A table costs him Rs. 400 and a chair Rs. 250. He can sell a table at a profit of Rs. 25 and a chair at a profit of Rs. 40. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

5. A small firm manufactures items A and B. The total number of items A and B it can manufacture a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit ? Solve the problem graphically.

6. A chemist requires 10, 12 and 12 units of chemicals A, B and C respectively for his analysis. A liquid product contains 5, 2, and 1 units of A, B and C respectively and it costs Rs. 3 per jar. A dry product contains 1, 2, and 4 units of A, B and C per carton and costs Rs. 2 per carton. How many of each should he purchase in order to minimize the cost and meet the requirement ?

7. A person wants to invest at most Rs. 18,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 4,000 in Bond A and at least Rs. 5,000 in Bond B. If the rate of interest on bond A is 9 % per annum and the rate of interest on bond B is 11 % per annum , how much money should he invest to earn maximum yearly income ?

8. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labourcost.

TOPIC 12
PROBABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Probability	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1
	(ii)Multiplication theorem on probability	**	**Article 13.3** SolvedExamples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2
	(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
	(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
	(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
	(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

SOME IMPORTANT RESULTS/CONCEPTS

** **Sample Space and Events** :

The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.

ϕ ($\subset$ S) is called an impossible event and

S($\subset$ S) is called a sure event.

** **Probability of an Event.**

(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is

defined as P(E) $\dfrac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$

it being assumed that the outcomes of the experiment in reference are equally likely.

(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = P(ϕ) = 0.

(iii) If $E_1, E_2, E_3, \ldots, E_k$ are mutually exclusive and exhaustive events associated with an experiment

(i.e. if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \ldots, k\}$ $i \neq j$), then

$$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_k) = 1.$$

(iv) $P(E) + P(E^C) = 1$

** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \text{ provided } P(F) \neq 0$$

** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$

$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

** Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \,.\, P(F)$.

** Bayes' Theorem : If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ andA is any event of nonzero probability, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \ldots, n$$

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^nC_x\, q^{n-x} p^x$, x = 0, 1,..., n. (q = 1 – p)

ASSIGNMENTS

(i) Conditional Probability

LEVEL I

1. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events.
2. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced.

LEVEL II

1.A dice is thrown twice and sum of numbers appearing is observed to be 6. what is the conditional probability that the number 4 has appeared at least once.

LEVEL III

1.If$P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{2}$, find $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$

(ii)Multiplication theorem on probability

LEVEL II

1.A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red.

2. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

LEVEL III

1.A class consists of 80 students; 25 of them are girls and 55 are boys, 10 of them are rich and the remaining poor; 20 of them are fair complexioned. what is the probability of selecting a fair complexioned rich girl.

2.Two integers are selected from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd.

(iii) Independent Events

LEVEL I

1. A coin is tossed thrice and all 8 outcomes are equally likely.
E : "The first throw results in head"   F : "The last throw results in tail"
Are the events independent ?

2. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent ?

3. If A and B are independent events, Find P(B) if $P(A \cup B) = 0.60$ and $P(A) = 0.35$.

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I

1. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag . Find the probability that the ball drawn is blue in colour.

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart.

3. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle.

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin.

5. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

LEVEL II

1. Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one-rupee coins and other contains 2 fifty-paise coins and 3 one-rupee coins.
2. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.
3. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 % . Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

LEVEL III

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from
(i) LONDON (ii) CLIFTON ?
2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease.
3. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III , there is one gold and one silver coin.
A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? **[CBSE 2011]**

(v) Random variables & probability distribution Mean & variance of random variables

LEVEL I

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades
2. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples.
3. A random variable X is specified by the following distribution

X	2	3	4
P(X)	0.3	0.4	0.3

Find the variance of the distribution.

LEVEL III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice.Find the probability distribution of the number of tails.

2.The sum of mean and variance of a binomial distribution for 5 trials be 1.8. Find the probability distribution.

3. The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

(vi) Bernoulli,s trials and Binomial Distribution

LEVEL II

1. If a die is thrown 5 times, what is the chance that an even number will come up exactly 3 times.
2. An experiment succeeds twice as often it fails. Find the probability that in the next six trials, there will be at least 4 success.

3. A pair of dice is thrown 200 times. If getting a sum 9 is considered a success, find the mean and variance of the number of success.

Questions for self evaluation

1. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

2. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of an experiment are performed. Find the probability that the event happens at least once.

3. A football match is either won, draw or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

4. A candidate has to reach the examination center in time. Probability of him going by bus ore scooter or by other means of transport are $\frac{3}{10},\frac{1}{10},\frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. But he reaches in time if he uses other mode of transport. He reached late at the centre. Find the probability that he traveled by bus.

5. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0, \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \end{cases}, \text{ k is a + ve constant.}$$

Find the mean and variance of the probability distribution. 1

6. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

7. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

8. Two cards are drawn simultaneously (or successively) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

ANSWERS

TOPIC 1 RELATIONS& FUNCTIONS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. R = { (3,5),(4,4),(5,3) },Domain = {3, 4, 5}, Range = {3, 4, 5}

2. Domain = {1, 2, 3,}, Range = {8, 9, 10}

(iii).One-one , onto & inverse of a function

LEVEL I

1. - f(x)   6. $\frac{1+x}{1-x}$

LEVEL II

2. $f^{-1}(x) = \frac{(4x+7)}{2}$ **3.**6   **5.**$f^{-1}(x) = \frac{(2x-5)}{3}$

(iv).Composition of function

LEVEL II

5.$f \circ f(x) = x$ 6 . $4x^2 - 12x + 9$

(v)Binary Operations

LEVEL I

5. 15   2. 4   3. e = 5   4.50

Questions for self evaluation

2. {1, 5, 9}   3. T_1 is related to T_3 6. $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

7. e = 0, $a^{-1} = \frac{a}{a-1}$ 8. Identity element (1 , 0), Inverse of (a , b) is $\left(\frac{1}{a}, \frac{-b}{a}\right)$

TOPIC 2 INVERSE TRIGONOMETRIC FUNCTION

1. Principal value branch Table

LEVEL I

1. $\frac{\pi}{6}$ 2. $-\frac{\pi}{6}$ 3. $-\frac{\pi}{3}$ 4. $\frac{3\pi}{4}$

LEVEL II

1. π 2. $\frac{\pi}{5}$ 3. $\frac{5\pi}{6}$

2. Properties of Inverse Trigonometric Functions

LEVEL I

1. 0   3. $\sqrt{2}$

LEVEL II

1. $\frac{1}{2}\tan^{-1} x$

LEVEL III

3. $\frac{1}{6}$　　4. $\frac{1}{4}$　　5. $\pm\frac{1}{\sqrt{2}}$

Questions for self evaluation

6. x　　7. $\pm\frac{1}{\sqrt{2}}$　　8. $\frac{1}{6}$

TOPIC 3 MATRICES & DETERMINANTS

1. Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. $1\times5,\ 5\times1$　　**2.** $\begin{bmatrix}1 & \frac{5}{2}\\ \frac{1}{2} & 2\\ 0 & \frac{3}{2}\end{bmatrix}$　　**3.** $\begin{bmatrix}-3 & -4 & 1\\ 1 & 1 & -1\end{bmatrix}$　　4**.** $2\times2,\ 3\times3$

LEVEL II

3.skew symmetric　　**4.** a = 8, b = 8

LEVEL III.

1. $\begin{bmatrix}1 & -1 & -1\\ 3 & -3 & 4\\ -3 & 2 & 0\end{bmatrix}$　　**2.** $\begin{bmatrix}3 & 1/2 & -5/2\\ 1/2 & -2 & -2\\ -5/2 & -2 & 2\end{bmatrix}+\begin{bmatrix}0 & -5/2 & -3/2\\ 5/2 & 0 & -3\\ 3/2 & 3 & 0\end{bmatrix}$

(ii). Cofactors &Adjoint of a matrix

LEVEL I

1. 46　　2. $\begin{bmatrix}3 & 1\\ -4 & 2\end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. $A^{-1} = -A$　　**2**. $A^{-1} = A$　　**3**. k = 17

LEVEL II

1. $\begin{bmatrix}-2/14 & -5/14\\ -4/14 & -3/14\end{bmatrix}$ **3. 0**

LEVEL III

1.x = 1, y = 2, z = 3.　　**2**. x = 3, ,y = -2, z = 1.　　**3.** AB = 6I, x = 2, y = - 1, z = 4

4.x = ½, y = -1, z = 1.　　**5.** $\begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$

(iv). To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. $\frac{1}{2}$ **2.** 27 **3.** 24 **4.** $\frac{4}{3}$

LEVEL II

1. 8 **2.** 49

LEVEL III

1. a = 3 **2.** 125

(v). Properties of Determinants

LEVEL I

1. x = 4 **2.** $a^2 + b^2 + c^2 + d^2$

LEVEL II

2. [Hint: Apply $C_1 \rightarrow -bC_3$ and $C_2 \rightarrow aC_3$]

LEVEL III

1a. 4 **1b.** $0, 0, 3a$ **1c.** $-\frac{a}{3}$

2. HINT $\Delta = \frac{-1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (a - b)^2]$

3. [Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively]

4.[Hint : Apply $R_1 \rightarrow R_1 + R_3$ and take common a + b + c]

5.Hint : Apply $R_1 \rightarrow aR_1$, $R_2 \rightarrow bR_2$, and $R_3 \rightarrow cR_3$]

6.[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively and then apply $R_1 \rightarrow R_1 + R_2 + R_3$]

Questions for self evaluation

4. $\begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & 1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$

6. x = 8, y = 8 and $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

8. $A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$, x = 2, y = 1, z = 3

9. $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, x = 3, y = – 2, z = –1

10. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

TOPIC 4 CONTINUITY AND DIFFRENTIABILITY

2.Continuity

LEVEL-I

1.Continuous   2.Not Continuous

LEVEL-II

2.3/4   3.$3a - 3b = 2$ 4. Not Continuous

LEVEL-III

1. 1 [Hint: Use 1-cos $2\theta = 2sin^2\theta$]2. 1 [Hint: Use $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]
3.a = 1/2, b = 4   4. K = 2

3.Differentiation

LEVEL- I

1.Not Differentiable   2. $\frac{2}{1+x^2}$

3. $\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left(\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{(6x+4)}{3x^2+4x+5}\right)$

LEVEL-II

1. 2logxsin(logx)^2/x   2.$\frac{1}{2(1+x^2)}$ 4.$-\frac{3}{2a}\left(\frac{1+t}{1-t}\right)^3$

LEVEL-III

1 .$\frac{x}{\sqrt{1-x^4}}$ [hint: Put $x^2 = \cos 2\theta$]   2. $\frac{1}{2}$[Hint: use $1 \pm sinx = (\cos\frac{x}{2} \pm \sin\frac{x}{2})^2$]

4.Logrithmic Differentiation

LEVEL-I

1.y′ = 1/(xlogxlog7)   2. $\frac{\cos(\log x)}{x}$

3.$\frac{dy}{dx} = \frac{1}{x(1+(\log x)^2)}$ [Hint: Use log(ex) = loge+logx=1+logx]

LEVEL-II

2. 2logxsin(logx)^2/x   3. $\frac{y\tan x+\log\cos y}{x\tan y+\log\cos x}$

LEVEL-III

$2\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$ 4. $x^{\cot x}(\cot x + x\log\sin x) + \frac{2x^2+14x+3}{(x^2+x+2)^2}$

(v) Parametric differentiation

2. $\frac{2\sqrt{2}}{a}$

Questions for self evaluation

1. k = 5 2.a = 3, b = 2, 4. $a = -\frac{3}{2}, c = \frac{1}{2}, b \in R$

5. $\left[\frac{dy}{dx}\right]_{\theta=\pi/4} = 1$ 6. $(\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x.\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$

7. $\frac{y-\sec^2 x}{1-x-2y}$ 9. [Hint: Put $x = \sin\theta$; $y = \sin\varphi$] 10. $\frac{y\tan x + \log\cos y}{x\tan y + \log cox}$

TOPIC 5 APPLICATIONS OF DERIVATIVES

1. Rate of change

LEVEL I 1. $\frac{27\pi}{8}(2x+1)^2$ 2.64 cm^2/min 3. 4.4 cm/sec

LEVEL II 1.(2 , 4) 2. 9 km/h 3.8 cm^2/sec

LEVEL III 1. (4 , 11) and $\left(-4, \frac{-31}{3}\right)$ 2. $\frac{4}{45}\pi$cm/sec 3. $\frac{1}{10}\pi$cm/sec

2. Increasing & decreasing functions

LEVEL I 3.(0, 3π/4) U (7π/4, 2π) and (3π/4, 7π/4)

LEVEL II 1. (0 ,π) 3. (0 , e) and (e, ∞)

LEVEL III 1. (- ½ , 0) U (½ , ∞) &(-∞ , -1/2) U (0,1/2)

3. Tangents &normals

LEVEL I 1. x + 3y - 8 = 0 & x + 3y + 8 =0 2. (0 , 0)
3. (1 , 0) & (1 , 4)

LEVEL II 1. $2x + 3my - 3am^4 - 2am^2 = 0$ 2. (3 , 45) & (–3 , 27)
3. x + 14y – 254 = 0 & x + 14y + 86 = 0

LEVEL III 1.80x – 40y – 103 = 0 3. $a^2 = b^2$[Hint: Use $m_1m_2 = -1$]

4. Approximations

LEVEL I 1. 5.03 2. 4.042 3.0.2867 4. 7.036

LEVEL II 1. 2.16 π cm

5 Maxima & Minima

LEVEL I 1.1 & 5 3. 12, 12

LEVEL II 2. $\frac{112}{\pi+4}$ cm , $\frac{28\pi}{\pi+4}$ cm. 3. Length = $\frac{20}{\pi+4}$ m , bredth $\frac{10}{\pi+4}$ m .

LEVEL III 1. $\frac{3\sqrt{3}}{4}$ ab 3. $\frac{4\left(6+\sqrt{3}\right)}{11}$ m , $\frac{30-6\sqrt{3}}{11}$

Questions for self evaluation

1. $\frac{1}{48\pi}$ cm/s 2. $b\sqrt{3}\text{cm}^2/\text{s}$ 3. $\uparrow$ in (– 2 , – 1) and $\downarrow$ in $(-\infty,-2)\cup(-1,\infty)$

4. $\uparrow$ in $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$ and $\downarrow$ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ 5. (0 , 0) , (1 , 2), (– 1, – 2)

6. (a) y – 2x – 3 = 0 , (b) 36y + 12x – 227 = 0 8.(i) 2.962 (ii) 2.962

10. $\frac{200}{7}$ m

TOPIC 6 INDEFINITE& DEFINITE INTEGRALS

(i) Integration by substitution

LEVEL I 1. $\tan(\log_e x) + C$ 2. $\frac{1}{m}e^{m\tan^{-1}x}+C$ 3. $e^{\sin^{-1}x}+C$

LEVEL II 1. $2\log_e\left|1+\sqrt{x}\right|+C$ 2. $\frac{1}{3}\sec^{-1}x^3+C$ 3. $\log_e\left|1-e^x\right|+C$

LEVEL III 1. $2\sqrt{\tan x}+C$ 2. $-\tan^{-1}(\cos x)+C$ 3. $\frac{\tan^2 x}{2}+\log_e|\tan x|+C$

(ii)) Application of trigonometric function in integrals

LEVEL I 1. $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x+C$ 2. $\frac{1}{2}\left[x+\frac{\sin 6x}{6}\right]+C$

3. $\frac{x}{4}+\frac{1}{4}\sin 6x+\frac{1}{16}\sin 4x+\frac{1}{8}\sin 2x+C$

LEVEL II 1. $\frac{1}{4}\sec^4 x+C$ OR $\frac{\tan^2 x}{2}+\frac{\tan^4 x}{4}+C$ 2. $\frac{2}{3}\sin 3x+2\sin x+C$

LEVEL III 1. $\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C$ 2. $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$

(iii) Integration using Standard results

LEVEL I 1. $\frac{1}{2}\log_e\left|x+\frac{1}{2}\sqrt{4x^2-9}\right|+C$ 2. $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+C$ 3. $\frac{1}{9}\tan^{-1}\left(\frac{3x+2}{3}\right)+C$

LEVEL II 1. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ 2. $\tan^{-1}(\sin x+2)+C$ 3. $\sin^{-1}\left(\frac{2x-1}{5}\right)+C$

LEVEL III $\quad$ 1. $\sin^{-1}\left(\dfrac{2x^2-1}{5}\right)+C$ $\quad$ 2. $x+\log\left|x^2-x+1\right|+\dfrac{2}{\sqrt{3}}\log\left|\dfrac{2x-1}{\sqrt{3}}\right|+C$

3. $\sqrt{x^2+5x+6}-\dfrac{1}{2}\log\left|\left(x+\dfrac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

4. $\sin^{-1}x+\sqrt{1-x^2}+C$ [Hint: Put x=cos2θ]

5. $6\sqrt{x^2-9x+20}+34\log\left|\left(\dfrac{2x-9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$

(iv) Integration using Partial Fraction

LEVEL I $\quad$ 1. $\dfrac{1}{3}\log(x+1)+\dfrac{5}{3}\log(x-2)+C$ $\quad$ 2. $\dfrac{1}{2}\log(x-1)-2\log(x-2)+\dfrac{3}{2}\log(x-3)+C$

3. $\dfrac{11}{4}\log\left(\dfrac{x+1}{x+3}\right)+\dfrac{5}{2(x+1)}+C$

LEVEL II $\quad$ 1. $x-11\log(x-1)+16\log(x-2)+C$ $\quad$ 2. $\dfrac{1}{4}\log x-\dfrac{1}{2x}+\dfrac{3}{4}\log(x+2)+C$

3. $\dfrac{3}{8}\log(x-1)-\dfrac{1}{2(x-1)}+\dfrac{5}{8}\log(x+3)+C$

LEVEL III $\quad$ 1. $\log(x+2)-\dfrac{1}{2}\log\left(x^2+4\right)+\tan^{-1}\dfrac{x}{2}$ $\quad$ 2. $\dfrac{\log(1-\cos x)}{6}+\dfrac{\log(1+\cos x)}{2}-\dfrac{2\log(1+2\cos x)}{3}+C$

3. $\dfrac{1}{3}\log(1+x)-\dfrac{1}{6}\log\left(1-x+x^2\right)+\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2x-1}{\sqrt{3}}\right)+C$ [Hint: Partial fractions]

(v) Integration by Parts

LEVEL I $\quad$ 1. $x.\tan x+\log\cos x+C$ $\quad$ 2. $x\log x-x+C$ 3. $e^x.\log\sec x+C$

LEVEL II $\quad$ 1. $x\sin^{-1}x+\sqrt{1-x^2}+C$ $\quad$ 2. $\dfrac{x^3}{3}\sin^{-1}x+\dfrac{\left(x^2+2\right)\sqrt{1-x^2}}{9}+C$

3. $-\sqrt{1-x^2}\sin^{-1}x+x+C$ $\quad$ 4. $2x\tan^{-1}x-\log\left(1+x^2\right)+C$

5. $\dfrac{1}{2}\left(\sec x.\tan x+\log(\sec x+\tan x)\right)+C$

LEVEL III $\quad$ 1. $\dfrac{x}{2}[\cos(\log x)+\sin(\log x)]+C$ $\quad$ 2. $\dfrac{e^x}{2+x}+C$ $\quad$ [Hint: $\int[e^x f(x)+f'(x)]dx= e^x f(x)+c$]

3. $\dfrac{x}{1+\log x}+C$

4. $e^x.\tan x+C$ $\quad$ 5. $\dfrac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+C$

(vi) Some Special Integrals

LEVEL I $\quad$ 1. $\dfrac{x\sqrt{4+x^2}}{2}+2\log\left|x+\sqrt{4+x^2}\right|+C$ $\quad$ 2. $\dfrac{x\sqrt{1-4x^2}}{2}+\dfrac{1}{4}\sin^{-1}2x+C$

LEVEL II

1. $\dfrac{(x+2)\sqrt{x^2+4x+6}}{2}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

2. $\dfrac{(x+2)\sqrt{1-4x-x^2}}{2}+\dfrac{5}{2}\sin^{-1}\left(\dfrac{x+2}{\sqrt{5}}\right)+C$

LEVEL III

1. $-\dfrac{1}{3}\left(1-x-x^2\right)^{3/2}+\dfrac{1}{8}(2x-1)\sqrt{1-x-x^2}+\dfrac{5}{16}\sin^{-1}\left(\dfrac{2x+1}{\sqrt{5}}\right)+C$

2. $\dfrac{1}{3}\left(x^2+x\right)^{3/2}-\dfrac{11}{8}(2x+1)\sqrt{x^2+x}+\dfrac{11}{16}\log\left[(2x+1)+2\sqrt{x^2+x}\right]+C$

(vii) Miscellaneous Questions

LEVEL II

1. $\dfrac{1}{2\sqrt{5}}\log\left|\dfrac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+C$

2. $\dfrac{1}{2\sqrt{2}}\tan^{-1}\left(\dfrac{3\tan x+1}{2\sqrt{2}}\right)+C$

3. $\dfrac{1}{2\sqrt{5}}\tan^{-1}\left(\dfrac{2\tan x}{\sqrt{5}}\right)+C$

4. $\dfrac{1}{6}\tan^{-1}\left(\dfrac{2\tan x}{3}\right)+C$

5. $\tan^{-1}\left(\tan^2 x\right)+C$ [Hint: divide Nr. and Dr. by x^2]

6. $\dfrac{2}{3}\tan^{-1}\left(\dfrac{5\tan\frac{x}{2}+4}{3}\right)+C$

LEVEL III

1. $-\dfrac{12}{13}x-\dfrac{5}{13}\log\left|3\cos x+2\sin x\right|+C$

2. $\dfrac{x}{2}-\dfrac{1}{2}\log\left|\cos x-\sin x\right|+C$

3. $x+\dfrac{1}{4}\log\left|\dfrac{x-1}{x+1}\right|-\dfrac{1}{2}\tan^{-1}x+C$

4. $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{x^2-1}{\sqrt{3}x}\right)+C$

5. $\dfrac{1}{2\sqrt{2}}\log\left|\dfrac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$

6. $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\tan x-1}{\sqrt{2\tan x}}\right)+\dfrac{1}{2\sqrt{2}}\left|\dfrac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+C$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\dfrac{1}{5}\log\ 6+\dfrac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

2. $\dfrac{64}{231}$

3. $\left[\log\dfrac{3}{2}-9\log\dfrac{5}{4}\right]$

LEVEL II

1. $5+\dfrac{5}{2}\left[\log\dfrac{3}{2}-9\log\dfrac{5}{4}\right]$

2. $\dfrac{e^2}{4}\left(e^2-2\right)$

(ii) Definite integrals as a limit of sum

LEVEL I 1. 6 2. 12

(iii) Properties of definite Integrals

LEVEL I

1. $\dfrac{\pi}{4}$

2. 1

3. $\dfrac{\pi}{4}$

LEVEL II 1. $\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 2. $\frac{\pi^2}{4}$ 3. $\frac{\pi^2}{4}$

4. $\frac{\pi}{12}$

LEVEL III 1. $\frac{\pi}{2}$ 2. $-\frac{\pi}{2}\log 2$ 3. $\frac{\pi}{8}\log 2$

(iv) Integration of modulus function

LEVEL III 1. $\frac{19}{2}$ 2. $\frac{11}{4}$ 3. 4

Questions for self evaluation

1. $\log\left|x^2+3x-18\right|-\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|+c$

2. $-3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

3. $\left[\frac{1}{8}\left(3x+2\sin 2x+\frac{\sin 4x}{4}\right)+c\right]$

4. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+c$

5. $\frac{18}{15}x+\frac{1}{25}\log\left|3\sin x+4\cos x\right|+c$

6. $x-\sqrt{1-x^2}\sin^{-1}x+c$

7. $\frac{64}{231}$

8. $\frac{3}{\pi}+\frac{1}{\pi^2}$

9. $-\frac{\pi}{2}\log 2$

10. 19/2

TOPIC 7 APPLICATIONS OF INTEGRATION

(i)Area under ***Simple Curves***

LEVEL I 1. 20π Sq. units 2. 6π Sq. units

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units 2. $\frac{32-8\sqrt{2}}{3}$ Sq. units

LEVEL III 1. $\frac{23}{6}$ Sq. units

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II 1. $5(\pi-2)$ Sq. units

(iv) Area of the region enclosed between***Circle and line***

LEVEL II 1. 4π Sq. units

LEVEL III 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units

(v) Area of the region enclosed between ***Circle and parabola***

LEVEL III 1. $2\left(4\pi-\frac{4\sqrt{3}}{3}\right)$ Sq. units 2. $\frac{4}{3}(8+3\pi)$ Sq. units

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III 1. $\left(\frac{8\pi}{3}-2\sqrt{3}\right)$ Sq. units

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II 1. 4 Sq. units

(viii) Area of triangle ***when vertices are given***

LEVEL III 1.4 Sq. units 2. $\frac{15}{2}$ Sq. units

(ix) Area of triangle ***when sides are given***

LEVEL III 1.8 Sq. units 2. 6 Sq. units

(x) ***Miscellaneous Questions***

LEVEL III 1. $\frac{1}{2}$ Sq. units 2. $\frac{1}{2}$ Sq. units

3. $\left(2-\sqrt{2}\right)$ Sq. units 4. 2 Sq. units

5. $\frac{64}{3}$ Sq. units 6. 9 Sq. units

Questions for self evaluation

1. $\frac{9}{8}$ sq. units 2. $\frac{1}{3}$ sq. units 3. $\frac{23}{6}$ sq. units

4. $\frac{3}{4}(\pi-2)$ sq. units 5. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units 6. $\frac{4}{3}(8+3\pi)$ sq. units

7. $\left(\frac{8\pi}{3}-2\sqrt{3}\right)$ sq. units 8. 4 sq. units

TOPIC 8 DIFFERENTIAL EQUATIONS

1. *Order and degree of a differential equation*

LEVEL I 1.order 2 degree 2

3. *Formation of differential equation*

LEVEL II 1. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

LEVEL III 1. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$ 2. $y^2-2xy\frac{dy}{dx}=0$ [Hint: $y^2=4ax$]

4. *Solution of differential equation by the method of separation of variables*

LEVEL II 1. $\log|1+y| = x + \frac{1}{2}x^2 + c$ 2. $e^y = \sin x + 1$

3. $y = \frac{1}{2}\log\left|1+x^2\right| + (\tan^{-1} x)^2 + c$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II 1. $\log|x| - \log|x-y| - \frac{y}{x} + c = 0$

LEVEL III 1. $cy = \log\frac{y}{x} - 1$ 2. $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$ 3. $y = ce^{\frac{x^3}{3y^3}}$

4. $y + \sqrt{x^2+y^2} = cx^2$ 5. $y = 3x^2 + cx$ 6. $y = \frac{x^3}{4} + \frac{c}{x}$

7. $y = -\frac{2}{3}x^2 + \frac{c}{x}$

6. *Linear Differential Equations*

LEVEL I 1. 1/x

LEVEL II 1. y=cosx+c cos2x 2. $\frac{y}{x+1} = \frac{1}{3}e^{3x} + c$ 3. xy = $\frac{x^2}{4}$ (2logx -1)+c

LEVEL III 1 $\tan\left(\frac{x+y}{2}\right) = x + c$ 2. $x = -y^2e^{-y} + cy^2$ 3. $-\frac{x}{y} = \log|x| + c$

4. $(x^2+1)^2 = -\tan^{-1}x + c$ [Hint: Use $\frac{dy}{dx} + Py = Q$] 5. $x = 2y^2$

Questions for self evaluation

1. Order 2, Degree not defined 2. $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

3. $x = \left(\tan^{-1} y - 1\right) + Ce^{-\tan^{-1} y}$ 4. $y + \sqrt{x^2+y^2} = Cx^2$

5. $y\log x = \frac{-2}{x}\left(1+\log|x|\right)+C$ 6. $y + 2x = 3x^2y$ [Hint: use $v = \frac{x}{y}$]

7. $2e^{\frac{x}{y}} + \log|y| = 2$

8. $y = x^2 - \frac{\pi^2}{4\sin x}$

TOPIC 9
VECTOR ALGEBRA

(i)Vector and scalars, Direction ratio and direction cosines &Unit vector

LEVEL I

1. $\frac{2}{\sqrt{17}}\hat{i} - \frac{3}{\sqrt{17}}\hat{j} - \frac{2}{\sqrt{17}}\hat{k}$ 2. $5\hat{i} - 10\hat{j} + 10\hat{k}$ 3. $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

4. $\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$ 5. $7\left(\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right)$

LEVEL II

1. $5\left(\frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}\right)$ 2. $\sqrt{3}$ 3. $2\hat{i} - \hat{j} + 4\hat{k}$

LEVEL III

1. 2 2. $P = \pm\frac{1}{\sqrt{3}}$ 3. Cosine of the angle with y –axis is $\frac{1}{2}$ 4. $P = \frac{2}{3}$

(ii)Position vector of a point and collinear vectors

LEVEL I

1. $4\hat{i} + \hat{j}$ 2 $\overrightarrow{CA} = -(3\hat{i} + 2\hat{j} + 7\hat{k})$

LEVEL II

1. $-3\hat{i} + 3\hat{k}$

(iii). Dot product of two vectors

LEVEL I

1. 1. $\vec{a} \cdot \vec{b} = 9$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{4}$

LEVEL II

1. $\vec{a} = 15\hat{i} - 27\hat{j} + 5\hat{k}$ 2. $\theta = \frac{\pi}{4}$ 3. $\lambda = 8$

LEVEL III

2. $\frac{\pi}{2}$ 3. (i) $\lambda = \frac{-40}{3}$ (ii) $\lambda = 6$ 4. $|\vec{x}| = 4$ 5. [Hint: Use $(\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = 0$]

7. $\vec{d} = 64\hat{\imath} - 2\hat{\jmath} - 28\hat{k}$ 9. $5\sqrt{2}$

(iv)Projection of a vector

LEVEL I

1. $\frac{8}{7}$ [Hint: Use projection of $\vec{a}$ *on* $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$] 2. 0 3. $\cos^{-1}\frac{5}{7}$

4. $\frac{60}{\sqrt{114}}$

LEVEL III

1. $[\frac{1}{\sqrt{2}}\hat{i}, \frac{1}{2}\hat{j}, \frac{1}{2}\hat{k}, \theta = \pi/3]$ 3. $\vec{\beta}_1 = \frac{1}{2}(3\hat{\imath} - \hat{\jmath})$, $\vec{\beta} = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$

(vii)Cross product of two vectors

LEVEL I

1. 12 2. $19\sqrt{2}$ 3. $|\vec{x}| = 9$ 4. $p = \frac{27}{2}$

LEVEL II

1. $\lambda = -3$ 3. $\theta = \frac{\pi}{6}$

LEVEL III

1. 1 [Hint: $\boldsymbol{cos^2\alpha + cos^2\beta + cos^2\gamma = 1}$] **2.** $\theta = \frac{\pi}{3}$ 3. $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

5. $\left(-\frac{\hat{i}}{2} - \hat{j} + \frac{\hat{k}}{2}\right) + \frac{5}{2}(\hat{i} + \hat{j})$

(viii)Area of a triangle & Area of a parallelogram

LEVEL I

1.$10\sqrt{3}$ Sq. units   2.$|\vec{a} \times \vec{b}|$ 3. $\frac{\sqrt{21}}{2}$ Squnits [Hint : Use area$\Delta = \frac{1}{2}\left|\overrightarrow{AB} \; x \; \overrightarrow{AC}\right|$]

LEVEL II

3. $\frac{\sqrt{61}}{2}$.

Questions for self evaluation

1. $\lambda=1$ 2. $5\sqrt{2}$ 3. $\frac{\pi}{2}$

4. $\hat{i} + 2\hat{j} + \hat{k}$ 5. $\pm\frac{3}{\sqrt{83}}\vec{i}, \mp\frac{5}{\sqrt{83}}\vec{j}, \mp\frac{7}{\sqrt{83}}\vec{k}$

TOPIC 10THREE DIMENSIONAL GEOMETRY

(i)Direction Ratios and Direction Cosines

LEVEL I

1.$\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},$ **2.** $\left[\text{Ans.}\frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right]$

3.$\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},$

LEVEL II

1. $-\frac{3}{7},-\frac{2}{7},\frac{6}{7}$ 2. < -3,-2,6>   3. $\frac{2}{\sqrt{14}},\frac{-1}{\sqrt{14}},\frac{3}{\sqrt{14}}$ 4. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL I

1. $\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$ 2.$\left[\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{6}\right]$

3.$\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2} = \lambda$ 4. $2x - 3y + z + 4 = 0$

(iii)plane and skew lines

LEVEL II

1. Lines are intersecting & point of intersection is (3,0,-1).

[Hint: For Coplanarity use $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ & a_1a_2 & a_3 \\ & b_1b_2 & b_3 \end{vmatrix}$

LEVEL III

2. Equation of AB is $\vec{r}$ = $(4\hat{\imath} + 5\hat{\jmath} + 10\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$.

3. Equation of BC is $\vec{r}$ = $(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$. Coordinates of D are (3,4,5).

(iv) Shortest distance between two lines

LEVEL II

1(a) $\frac{3\sqrt{2}}{2}$ *units*, 4.$\frac{8}{\sqrt{29}}$

1(b) $\frac{3}{\sqrt{19}}$ units

5. $\frac{1}{\sqrt{3}}$

3. 0

6. Vector equation $\vec{r}$ = $(3\hat{\imath} - 4\hat{k}) + \lambda(5\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$ and distance = 7.75 units

(v)Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1.x = 0 2. 12x + 4y + 3z = 12 3.$\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$ 4. (a) $x + y - z = 2$ (b) $2x + 3y - 4z = 1$

LEVEL II

1. $2x + 3y - z = 20$ 2. $\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}\right) = 7$

3.$[r - (\hat{\imath} - 2\hat{k})] \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0;\ x + y - z = 3$

(vi) Angle Between (i)Two lines(ii)Two planes(iii)Line & plane

LEVEL-I

1. 60^0 2. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ 3. $\lambda = -3$ 4.$\cos^{-1}\frac{15}{\sqrt{731}}$ 5.$\sin^{-1}\frac{8}{21}$

LEVEL-II

1. p = –3

(vii) *Distance of a point from a plane*

LEVEL-I

1. 1/3 2. (0, 0, 0) 3. $\frac{13}{7}$

4. $\frac{1}{3}$ 5. [$\sqrt{b^2+c^2}$]

LEVEL-II

1. (4 , 3, 7) 2. 6 units 3. $\frac{3\sqrt{34}}{17}$ 4. 13

LEVEL-III

1. Foot of perpendicular (-1,4,3), Image (-3,5,2), Distance = $\sqrt{6}$ units

2. 3x – 4y + 3z -19 = 0

3. x + y – z – 2 = 0 , $\frac{2}{\sqrt{3}}units$, $\frac{1}{\sqrt{3}}$ units.

(viii).Equation of a plane through the intersection of two planes

LEVEL-II

1. x – y + 3z – 2 = 0 , $\frac{2\sqrt{11}}{11}$ 2. Ans. 51x + 15y – 50z + 173 = 0

3. 5x – 4y –z = 7

LEVEL-III

1. x – 2y + z = 0 3. x– 19y – 11z = 0 4. $\frac{x-2}{-6} = \frac{y-4}{-3} = \frac{z+1}{-2}$

(ix) Foot of perpendicular and image with respect to a line and plane

LEVEL-II

1. (1, -2, 7) 2. (3, 5, 9) 3. Image of the point = (0,-1,-3)

LEVEL-III

1. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$ 2. $\left(\frac{39}{7},\frac{-6}{7},\frac{-37}{7}\right)$ 3. 12x - 4y + 3z = 169 4. (-1,4,-1)

Questions for self evaluation

1. 17x + 2y – 7z = 12

2. $\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$,

3. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

4.ZERO

5. $\left(\frac{39}{7}, \frac{-6}{7}, \frac{-37}{7}\right)$

8. [Hint: second line can also be written as $-\frac{(x-4)}{5} = \frac{(y-1)}{2} = \frac{(z-0)}{1}$]

TOPIC 11LINEAR PROGRAMMING

(i) LPP and its Mathematical Formulation

LEVEL I

1. Z = $50x + 70y$, $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

1. Minimum Z = – 12 at (4, 0),

2. Maximum Z = $\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

3. Minimum Z = 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

(iii) Diet Problem

LEVEL II

1. Least cost = Rs.110 at x = 5 and y = 30

2. Minimum cost = Rs.6 at x = 400 and y = 200

(iv) Manufacturing Problem

LEVEL II

1.Maximum profit is Rs. 120 when 12 units of A and 6 units of B are produced

2. For maximum profit, 25 units of product A and 125 units of product B are produced and sold.

(v) Allocation Problem

LEVEL II

1.Maximum annual income = Rs. 6,200 on investment of Rs. 40,000 on Bond A and

Rs. 30,000 on Bond B.

2. A should run for 60 days and B for 30 days.

LEVEL III

1.For maximum profit, 62 executive class tickets and 188 economy class ticket should be sold.

(vi) Transportation Problem

LEVEL III

1. Minimum transportation cost is Rs. 400 when 10, 0 and 50 packets are transported from factory at A and 30, 40 and 0 packets are transported from factory at B to the agencies at P, Q and R respectively.

Questions for self evaluation

1. Minimum 155 at (0 , 5).

2. Minimum value is 5 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

3. Maximum is Rs 4.60 at (0.6 , 0.4)

4.Maximum is Rs.800 at (0 , 20)

5.8 items of type A and 16 items of type B

6.1 jar of liquid and 5 cartons of dry product.

7.Rs.4,000 in Bond A and Rs.14,000 in Bond B

8. Minimum cost Rs.1350 at (5 , 3)

TOPIC 12PROBABILITY

(i) Conditional Probability

LEVEL I 1. 0 2. $\frac{5}{14}$

LEVEL II 1. $\frac{2}{5}$

LEVEL III 1. $\frac{3}{4}$ and $\frac{3}{5}$

(ii)Multiplication theorem on probability

LEVEL II 1. $\frac{8}{65}$ 2.**(i)** $\frac{13}{21}$ (ii) $\frac{10}{21}$ [Hint : p(x>=1) = 1 – P(x<0)

LEVEL III 1. $\frac{5}{512}$ 2. $\frac{3}{5}$

(iii) Independent Events

LEVEL I 1.Yes 2.Yes [check: P(A∩B) = P(A).P(B)] 3. $\frac{5}{13}$

–

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I 1. $\frac{93}{154}$ 2. $\frac{11}{50}$ 3. $\frac{3}{4}$ 4. $\frac{19}{42}$ 5. 0.27

LEVEL II 1. $\frac{1}{2}$ 2. $\frac{2}{3}$ 3. $\frac{3}{7}$

LEVEL III 1.(i) $\frac{12}{17}$ (ii) $\frac{5}{17}$ 2.0.15 3. $\frac{2}{3}$

(v) Random variables & probability distribution , Mean & variance of random variables

LEVEL I 1.

X	0	1	2
P(X)	9/16	6/16	1/16

2.

X	0	1	2	3
P(X)	28/57	24/57	24/285	1/285

3. 0.6

LEVEL III 1.

X	0	1	2
P(X)	9/16	6/16	1/16

2.. $\left(\frac{4}{5}+\frac{1}{5}\right)^5$ 3. $\frac{65}{81}$

(vi) Bernoulli's trials and Binomial Distribution

LEVEL II 1. $\frac{5}{16}$ 2. $\frac{496}{729}$ 3. $\frac{200}{9}$, $\frac{1600}{81}$ [Hint: mean =np, variance=npq]

Questions for self evaluation

1. $\frac{1}{4}$ 2.0.784 3. $\frac{1}{9}$

4. $\frac{9}{13}$

5. $\frac{19}{8}, \frac{47}{64}$

6. $\frac{625}{23328}$

7. $\frac{11}{243}$

8. 1 and 1.47

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   Author: Raisinghania, M.D, Shanti Narayan

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Some useful weblinks

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CLASS XII

MATHEMATICS

	Units	Weightage (Marks)
(i)	Relations and Functions	10
(ii)	Algebra	13
(iii)	Calculus	44
(iv)	Vector and Three Dimensional Geometry	17
(v)	Linear Programming	06
(vi)	Probability	10
		Total : 100

Unit I : RELATIONS AND FUNCTIONS

1. Relations and Functions **(10 Periods)**

Types of Relations : Reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions **(12 Periods)**

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Unit II : ALGEBRA

1. Matrices **(18 Periods)**

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants **(20 Periods)**

Determinant of a square matrix (up to 3 × 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III : CALCULUS

1. Continuity and Differentiability **(18 Periods)**

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivatives. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's mean Value Theorems (without proof) and their geometric interpretations.

2. Applications of Derivatives **(10 Periods)**

Applications of Derivatives : Rate of change, increasing/decreasing functions, tangents and normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Sample problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals **(20 Periods)**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type to be evaluated.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$\int \frac{(px+q)}{ax^2+bx+c}dx, \int \frac{(px+q)}{\sqrt{ax^2+bx+c}}dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2-a^2}\, dx,$

$\int \sqrt{ax^2+bx+c}\, dx$ and $\int (px+q)\sqrt{ax^2+bx+c}\, dx$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals (10 Periods)

Application in finding the area under simple curves, especially lines, area of circles/parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

5. Differential Equations (10 Periods)

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type :

$\frac{dy}{dx} + p(x)y = q(x)$, where $p(x)$ and $q(x)$ are function of x.

Unit IV : VECTORS AND THREE-DIMENSIONAL GEOMETRY

1. Vectors (12 Periods)

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors. Scalar triple product of vectors.

2. Three-Dimensional Geometry (12 Periods)

Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (*i*) two lines, (*ii*) two planes, (*iii*) a line and a plane. Distance of a point from a plane.

Unit V : LINEAR PROGRAMMING

(12 Periods)

1. **Linear Programming :** Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit VI : PROBABILITY

1. **Probability** **(18 Periods)**

   Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

CONTENTS

CHAPTER 1

RELATIONS AND FUNCTIONS

IMPORTANT POINTS TO REMEMBER

- Relation R from a set A to a set B is subset of $A \times B$.
- $A \times B = \{(a, b) : a \in A, b \in B\}$.
- If $n(A) = r$, $n(B) = s$ from set A to set B then $n(A \times B) = rs$.

  and no. of relations $= 2^{rs}$
- ϕ is also a relation defined on set A, called the void (empty) relation.
- $R = A \times A$ is called universal relation.
- **Reflexive Relation :** Relation R defined on set A is said to be reflexive iff $(a, a) \in R \ \forall \ a \in A$
- **Symmetric Relation :** Relation R defined on set A is said to be symmetric iff $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b, \in A$
- **Transitive Relation :** Relation R defined on set A is said to be transitive if $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in R$
- **Equivalence Relation :** A relation defined on set A is said to be equivalence relation iff it is reflexive, symmetric and transitive.
- **One-One Function :** $f : A \to B$ is said to be one-one if distinct elements in A has distinct images in B. *i.e.* $\forall \ x_1, x_2 \in A$ *s.t.* $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

  OR

  $\forall \ x_1, x_2 \in A$ s.t. $f(x_1) = f(x_2)$

  $\Rightarrow \ x_1 = x_2$

  One-one function is also called injective function.

- Onto function (surjective) : A function $f : A \to B$ is said to be onto iff $R_f = B$ *i.e.* $\forall \; b \in B$, there exist $a \in A$ s.t. $f(a) = b$
- A function which is not one-one is called many-one function.
- A function which is not onto is called into.
- **Bijective Function :** A function which is both injective and surjective is called bijective.
- **Composition of Two Function :** If $f : A \to B$, $g : B \to C$ are two functions, then composition of f and g denoted by gof is a function from A to C given by, $(gof)(x) = g(f(x)) \;\forall\; x \in A$

  Clearly gof is defined if Range of $f \subset$ domain of g. Similarly fog can be defined.
- **Invertible Function :** A function $f : X \to Y$ is invertible iff it is bijective.

  If $f : X \to Y$ is bijective function, then function $g : Y \to X$ is said to be inverse of f iff $fog = I_y$ and $gof = I_x$

  when I_x, I_y are identity functions.
- g is inverse of f and is denoted by f^{-1}.
- **Binary Operation :** A binary operation '*' defined on set A is a function from $A \times A \to A$. $*(a, b)$ is denoted by $a * b$.
- Binary operation * defined on set A is said to be commutative iff

  $$a * b = b * a \;\forall\; a, b \in A.$$
- Binary operation * defined on set A is called associative iff $a * (b * c) = (a * b) * c \;\forall\; a, b, c \in A$
- If * is Binary operation on A, then an element $e \in A$ is said to be the identity element iff $a * e = e * a \;\forall\; a \in A$
- Identity element is unique.
- If * is Binary operation on set A, then an element b is said to be inverse of $a \in A$ iff $a * b = b * a = e$
- Inverse of an element, if it exists, is unique.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If A is the set of students of a school then write, which of following relations are. (Universal, Empty or neither of the two).

   $R_1 = \{(a, b) : a, b$ are ages of students and $|a - b| \geq 0\}$

   $R_2 = \{(a, b) : a, b$ are weights of students, and $|a - b| < 0\}$

   $R_3 = \{(a, b) : a, b$ are students studying in same class$\}$

2. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as $R = \{(a, b) : b = a + 1\}$ reflexive?

3. If R, is a relation in set N given by

   $R = \{(a, b) : a = b - 3, b > 5\}$,

   then does elements $(5, 7) \in$ R?

4. If $f : \{1, 3\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 2, 3, 4\}$ be given by

   $f = \{(1, 2), (3, 5)\}$, $g = \{(1, 3), (2, 3), (5, 1)\}$

   Write down gof.

5. Let $g, f : R \rightarrow R$ be defined by

   $g(x) = \dfrac{x+2}{3}, f(x) = 3x - 2.$ Write fog.

6. If $f : R \rightarrow R$ defined by

   $f(x) = \dfrac{2x-1}{5}$

   be an invertible function, write $f^{-1}(x)$.

7. If $f(x) = \dfrac{x}{x+1} \forall x \neq -1$, Write $fo\,f(x)$.

8. Let * is a Binary operation defined on R, then if

   (i) $a * b = a + b + ab$, write $3 * 2$

(ii) $a * b = \frac{(a+b)^2}{3}$, Write $(2*3)*4$.

9. If $n(A) = n(B) = 3$, Then how many bijective functions from A to B can be formed?

10. If $f(x) = x + 1$, $g(x) = x - 1$, Then (gof) (3) = ?

11. Is $f : N \to N$ given by $f(x) = x^2$ is one-one? Give reason.

12. If $f : R \to A$, given by

$f(x) = x^2 - 2x + 2$ is onto function, find set A.

13. If $f : A \to B$ is bijective function such that $n(A) = 10$, then $n(B) = ?$

14. If $n(A) = 5$, then write the number of one-one functions from A to A.

15. $R = \{(a, b) : a, b \in N, a \neq b$ and a divides $b\}$. Is R reflexive? Give reason?

16. Is $f : R \to R$, given by $f(x) = |x - 1|$ is one-one? Give reason?

17. $f : R \to B$ given by $f(x) = \sin x$ is onto function, then write set B.

18. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

19. If '*' is a binary operation on set Q of rational numbers given by $a * b = \frac{ab}{5}$ then write the identity element in Q.

20. If * is Binary operation on N defined by $a * b = a + ab \;\forall\; a, b \in N$. Write the identity element in N if it exists.

SHORT ANSWER TYPE QUESTIONS (4 Marks)

21. Check the following functions for one-one and onto.

(a) $f : R \to R, \quad f(x) = \frac{2x-3}{7}$

(b) $f : R \to R$, $f(x) = |x + 1|$

(c) $f : R - \{2\} \to R$, $f(x) = \frac{3x-1}{x-2}$

(d) $f : R \to [-1, 1]$, $f(x) = \sin^2 x$

22. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b$ = H.C.F. of a and b. Write the operation table for the operation *****.

23. Let $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function given by $f(x) = \frac{4x}{3x+4}$. Show that f is invertible with $f^{-1}(x) = \frac{4x}{4-3x}$.

24. Let R be the relation on set $A = \{x : x \in Z, 0 \le x \le 10\}$ given by $R = \{(a, b) : (a - b)$ is multiple of $4\}$, is an equivalence relation. Also, write all elements related to 4.

25. Show that function $f : A \to B$ defined as $f(x) = \frac{3x + 4}{5x - 7}$ where $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ is invertible and hence find f^{-1}.

26. Let ***** be a binary operation on Q. Such that $a * b = a + b - ab$.

    (i) Prove that ***** is commutative and associative.

    (ii) Find identify element of ***** in Q (if it exists).

27. If * is a binary operation defined on $R - \{0\}$ defined by $a * b = \frac{2a}{b^2}$, then check ***** for commutativity and associativity.

28. If $A = N \times N$ and binary operation ***** is defined on A as

    $(a, b) * (c, d) = (ac, bd)$.

    (i) Check ***** for commutativity and associativity.

    (ii) Find the identity element for ***** in A (If it exists).

29. Show that the relation R defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

30. Let * be a binary operation on set Q defined by $a * b = \frac{ab}{4}$, *show that*

    (i) 4 is the identity element of * on Q.

(ii) Every non zero element of Q is invertible with

$$a^{-1} = \frac{16}{a}, \quad a \in Q - \{0\}.$$

31. Show that $f : R_+ \rightarrow R_+$ defined by $f(x) = \frac{1}{2x}$ is bijective where R_+ is the set of all non-zero positive real numbers.

32. Consider $f : R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ show that f is invertible with $f^{-1} = \frac{\sqrt{x+6}-1}{3}$.

33. If '*' is a binary operation on R defined by $a * b = a + b + ab.$ Prove that * is commutative and associative. Find the identify element. Also show that every element of R is invertible except –1.

34. If $f, g : R \rightarrow R$ defined by $f(x) = x^2 - x$ and $g(x) = x + 1$ find (fog) (x) and (gof) (x). Are they equal?

35. $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, find $f^{-1}(x)$.

36. $f : R \rightarrow R, g : R \rightarrow R$ given by $f(x) = [x]$, $g(x) = |x|$ then find

$$(fog)\left(\frac{-2}{3}\right) \text{ and } (gof)\left(\frac{-2}{3}\right).$$

ANSWERS

1. R_1 : is universal relation.

   R_2 : is empty relation.

   R_3 : is neither universal nor empty.

2. No, R is not reflexive.

3. $(5, 7) \notin R$

4. $gof = \{(1, 3), (3, 1)\}$

5. $(fog)(x) = x \ \forall \ x \in R$

6. $f^{-1}(x) = \dfrac{5x+1}{2}$

7. $(fof)(x) = \dfrac{x}{2x+1}, x \neq -\dfrac{1}{2}$

8. (i) 3 * 2 = 11

   (ii) $\dfrac{1369}{27}$

9. 6

10. 3

11. Yes, f is one-one $\because \forall\, x_1, x_2 \in N \Rightarrow x_1^2 = x_2^2$.

12. $A = [1, \infty)$ because $R_f = [1, \infty)$

13. $n(B) = 10$

14. 120.

15. No, R is not reflexive $\because (a, a) \notin R \;\forall\; a \in N$

16. f is not one-one functions

    $\because\ f(3) = f(-1) = 2$

    $3 \neq -1$ *i.e.* distinct element has same images.

17. $B = [-1, 1]$

19. $e = 5$

20. Identity element does not exist.

21. (a) Bijective

    (b) Neither one-one nor onto.

    (c) One-one, but not onto.

    (d) Neither one-one nor onto.

22.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

24. Elements related to 4 are 0, 4, 8.

25. $f^{-1}(x) = \dfrac{7x+4}{5x-3}$

26. 0 is the identity element.

27. Neither commutative nor associative.

28. (i) Commutative and associative.

    (ii) (1, 1) is identity in $N \times N$

33. *0* is the identity element.

34. $(fog)\ (x) = x^2 + x$

    $(gof)\ (x) = x^2 - x + 1$

    Clearly, they are unequal.

35. $f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

36. $(fog)\left(\dfrac{-2}{3}\right) = 0$

    $(gof)\left(\dfrac{-2}{3}\right) = 1$

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

IMPORTANT POINTS

- $\sin^{-1} x$, $\cos^{-1} x$, ... etc., are angles.
- If $\sin\theta = x$ and $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ then $\theta = \sin^{-1}x$ etc.
-

Function	***Domain***	***Range (Principal Value Branch)***
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\mathrm{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- $\sin^{-1} (\sin x) = x \ \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

  $\cos^{-1} (\cos x) = x \ \forall \ x \in [0, \pi]$ etc.
- $\sin (\sin^{-1}x) = x \ \forall \ x \in [-1, 1]$

  $\cos (\cos^{-1}x) = x \ \forall \ x \in [-1, 1]$ etc.

- $\sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) \ \forall x \in [-1, 1]$

  $\tan^{-1}x = \cot^{-1}(1/x) \ \forall \ x > 0$

  $\sec^{-1}x = \cos^{-1}(1/x), \ \forall \ |x| \geq 1$

- $\sin^{-1}(-x) = -\sin^{-1}x \ \forall \ x \in [-1, 1]$

  $\tan^{-1}(-x) = -\tan^{-1}x \ \forall \ x \in R$

  $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \ \forall \ |x| \geq 1$

- $\cos^{-1}(-x) = \pi - \cos^{-1}x \ \forall \ x \in [-1, 1]$

  $\cot^{-1}(-x) = \pi - \cot^{-1}x \ \forall \ x \in - R$

  $\sec^{-1}(-x) = \pi - \sec^{-1}x \ \forall \ |x| \geq 1$

- $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \quad x \in [-1, 1]$

  $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \ \forall \ x \in R$

  $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \ \forall \ |x| \geq 1$

- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); \quad xy < 1.$

- $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right); \quad xy > -1.$

- $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$

  $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \leq 1,$

  $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0.$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i) $\sin^{-1}\left(-\sqrt{3}/2\right)$ (ii) $\cos^{-1}\left(\sqrt{3}/2\right)$.

(iii) $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ (iv) cosec^{-1} (– 2).

(v) $\cot^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$. (vi) $\sec^{-1}$ (– 2).

(vii) $\sin^{-1}\left(\dfrac{-\sqrt{3}}{2}\right) + \cos^{-1}\left(\dfrac{-1}{2}\right) + \tan^{-1}\left(-1/\sqrt{3}\right)$

2. What is value of the following functions (using principal value).

(i) $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$. (ii) $\sin^{-1}\left(-\dfrac{1}{2}\right) - \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$.

(iii) $\tan^{-1}(1) - \cot^{-1}(-1)$. (iv) $\text{cosec}^{-1}\left(\sqrt{2}\right) + \sec^{-1}\left(\sqrt{2}\right)$.

(v) $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$.

(vi) $\sin^{-1}\left(\sin\dfrac{4\pi}{5}\right)$. (vii) $\tan^{-1}\left(\tan\dfrac{5\pi}{6}\right)$.

(viii) $\text{cosec}^{-1}\left(\text{cosec}\,\dfrac{3\pi}{4}\right)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

3. Show that $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \dfrac{\pi}{4} + \dfrac{x}{2}$. $x \in [0, \pi]$

4. Prove

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) - \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{\pi}{4} \qquad x \in (0, \pi/2).$$

5. Prove $\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) = \sin^{-1}\frac{x}{a} = \cos^{-1}\left(\frac{\sqrt{a^2 - x^2}}{a}\right).$

6. Prove

$$\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right] + \tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right] = \tan^{-1}\left(\frac{300}{161}\right).$$

7. Prove $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2.$

8. Solve $\cot^{-1}2x + \cot^{-1}3x = \frac{\pi}{4}.$

9. Prove that $\tan^{-1}\left(\frac{m}{n}\right) - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}, m, n > 0$

10. Prove that $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right] = \frac{x+y}{1-xy}$

11. Solve for x, $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{-2x}{1-x^2}\right) = \frac{2\pi}{3}$

12. Prove that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

13. Solve for x, $\tan\left(\cos^{-1}x\right) = \sin\left(\tan^{-1}2\right);\ x > 0$

14. Prove that $2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{32}{43}\right)$

15. Evaluate $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)\right]$

16. Prove that $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = \tan^{-1}\left(\frac{a}{b}\right) - x$

17. Prove that

$$\cot\left\{\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)\right\} + \cos^{-1}\left(1-2x^2\right) + \cos^{-1}\left(2x^2-1\right) = \pi, \quad x > 0$$

18. Prove that $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0$ where a, b, $c > 0$

19. Solve for x, $2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

20. Express $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$ in simplest form.

21. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$, then

prove that $a + b + c = abc$

22. If $\sin^{-1}x > \cos^{-1}x$, then x belongs to which interval?

ANSWERS

1. (i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{6}$ (iii) $\frac{-\pi}{6}$ (iv) $\frac{-\pi}{6}$

(v) $\frac{\pi}{3}$ (vi) $\frac{2\pi}{3}$ (vii) $\frac{\pi}{6}$.

2. (i) 0 (ii) $\frac{-\pi}{3}$ (iii) $-\frac{\pi}{2}$ (iv) $\frac{\pi}{2}$

(v) π (vi) $\frac{\pi}{5}$ (vii) $\frac{-\pi}{6}$ (viii) $\frac{\pi}{4}$.

8. 1

11. $\tan\frac{\pi}{12} = 2-\sqrt{3}$

13. $\frac{\sqrt{5}}{3}$

15. $\sqrt{\frac{\sqrt{11}-3}{3+\sqrt{11}}}$

19. $x = \frac{\pi}{4}$.

20 $\sin^{-1} x - \sin^{-1} \sqrt{x}$.

22. $\left(\frac{1}{\sqrt{2}}, 1\right]$

21. ***Hint:*** Let $\tan^{-1} a = \alpha$

$\tan^{-1} b = \beta$

$\tan^{-1} c = \gamma$

then given, $\alpha + \beta + \gamma = \pi$

$\therefore \quad \alpha + \beta = \pi - \gamma$

take tangent on both sides,

$\tan(\alpha + \beta) = \tan(\pi - \gamma)$

CHAPTER 3 & 4

MATRICES AND DETERMINANTS

POINTS TO REMEMBER

- **Matrix :** A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements of the matrix.
- **Order of Matrix :** A matrix having 'm' rows and 'n' columns is called the matrix of order mxn.
- **Square Matrix :** An mxn matrix is said to be a square matrix of order n if $m = n$.
- **Column Matrix :** A matrix having only one column is called a column matrix i.e. $A = [aij]_{mx1}$ is a column matrix of order mx1.
- **Row Matrix :** A matrix having only one row is called a row matrix i.e. $B = [bij]_{1xn}$ is a row matrix of order 1xn.
- **Zero Matrix :** A matrix having all the elements zero is called zero matrix or null matrix.
- **Diagonal Matrix :** A square matrix is called a diagonal matrix if all its non diagonal elements are zero.
- **Scalar Matrix :** A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.
- **Identity Matrix :** A scalar matrix in which each diagonal element is 1, is called an identity matrix or a unit matrix. It is denoted by I.

$\therefore \qquad I = [e_{ij}]_{n \times n}$

where, $\qquad e_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$

- **Transpose of a Matrix :** If $A = [a_{ij}]_{m \times n}$ be an $m \times n$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by A' or A^T.

  Properties of the transpose of a matrix.

  (i) $(A')' = A$ (ii) $(A + B)' = A' + B'$

  (iii) $(kA)' = kA'$, k is a scalar (iv) $(AB)' = B'A'$

- **Symmetrix Matrix :** A square matrix $A = [a_{ij}]$ is symmetrix if $a_{ij} = a_{ji}$ $\forall$ i, j. Also a square matrix A is symmetrix if $A' = A$.

- **Skew Symmetrix Matrix :** A square matrix $A = [a_{ij}]$ is skew-symmetrix, if $a_{ij} = - a_{ji}$ $\forall$ i, j. Also a square matrix A is skew - symmetrix, if $A' = - A$.

- **Determinant :** To every square matrix $A = [a_{ij}]$ of order $n \times n$, we can associate a number (real or complex) called determinant of A. It is denoted by det A or $|A|$.

  Properties

  (i) $|AB| = |A|\ |B|$

  (ii) $|kA|_{n \times n} = k^n\ |A|_{n \times n}$ where k is a scalar.

  Area of triangles with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

  The points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

- **Adjoint of a Square Matrix *A*** is the transpose of the matrix whose elements have been replaced by their cofactors and is denoted as *adj A*.

$$\text{Let} \quad A = [a_{ij}]_{n \times n}$$

$$adj\ A = [A_{ji}]_{n \times n}$$

Properties

(i) $A(adj\ A) = (adj\ A)\ A = |A|\ I$

(ii) If A is a square matrix of order n then $|adj\ A| = |A|^{n-1}$

(iii) $adj\ (AB) = (adj\ B)\ (adj\ A)$.

[***Note :*** Correctness of *adj* A can be checked by using A.(adj A) = (adj A) . A = |A| I]

Singular Matrix : A square matrix is called singular if $|A| = 0$, otherwise it will be called a non-singular matrix.

Inverse of a Matrix : A square matrix whose inverse exists, is called invertible matrix. Inverse of only a non-singular matrix exists. Inverse of a matrix A is denoted by A^{-1} and is given by

$$A^{-1} = \frac{1}{|A|}.\,adj.\,A$$

Properties

(i) $AA^{-1} = A^{-1}A = I$

(ii) $(A^{-1})^{-1} = A$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $(A^{T})^{-1} = (A^{-1})^{T}$

- **Solution of system of equations using matrix :**

If $AX = B$ is a matrix equation then its solution is $X = A^{-1}B$.

(i) If $|A| \neq 0$, system is consistent and has a unique solution.

(ii) If $|A| = 0$ and $(adj\ A)\ B \neq 0$ then system is inconsistent and has no solution.

(iii) If $|A| = 0$ and $(adj\ A)\ B = 0$ then system is either consistent and has infinitely many solutions or system is inconsistent and has no solution.

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y.

2. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, find AB.

3. Find the value of $a_{23} + a_{32}$ in the matrix $A = [a_{ij}]_{3 \times 3}$
   where $a_{ij} = \begin{cases} |2i - j| & \text{if } i > j \\ -i + 2j + 3 & \text{if } i \le j \end{cases}$.

4. If B be a 4×5 type matrix, then what is the number of elements in the third column.

5. If $A = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ find $3A - 2B$.

6. If $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -6 \end{bmatrix}$ find $(A + B)'$.

7. If $A = [1 \quad 0 \quad 4]$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ find AB.

8. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric matrix, then find x.

9. For what value of x the matrix $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & x+5 \end{bmatrix}$ is skew symmetrix matrix.

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$ where P is symmetric and Q is skew-symmetric matrix, then find the matrix Q.

11. Find the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

12. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, find x.

13. For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse.

14. If $A = \begin{bmatrix} \sin 30^\circ & \cos 30^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$, what is $|A|$.

15. Find the cofactor of a_{12} in $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

16. Find the minor of a_{23} in $\begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$.

17. Find the value of P, such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & P \end{bmatrix}$ is singular.

18. Find the value of x such that the points (0, 2), (1, x) and (3, 1) are collinear.

19. Area of a triangle with vertices (k, 0), (1, 1) and (0, 3) is 5 unit. Find the value (s) of k.

20. If A is a square matrix of order 3 and $|A| = -2$, find the value of $|-3A|$.

21. If $A = 2B$ where A and B are square matrices of order 3×3 and $|B| = 5$, what is $|A|$?

22. What is the number of all possible matrices of order 2×3 with each entry 0, 1 or 2.

23. Find the area of the triangle with vertices (0, 0), (6, 0) and (4, 3).

24. If $\begin{vmatrix} 2x & 4 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$, find x.

25. If $A = \begin{bmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{bmatrix}$, write the value of det *A.*

26. If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that $|A| = -15$, find $a_{11}\, C_{21} + a_{12} C_{22}$ where C_{ij} is cofactors of a_{ij} in $A = [a_{ij}]$.

27. If A is a non-singular matrix of order 3 and $|A| = -3$ find $|adj\ A|$.

28. If $A = \begin{bmatrix} 5 & -3 \\ 6 & 8 \end{bmatrix}$ find $(adj\ A)$

29. Given a square matrix A of order 3×3 such that $|A| = 12$ find the value of $|A\ adj\ A|$.

30. If A is a square matrix of order 3 such that $|adj\ A| = 8$ find $|A|$.

31. Let A be a non-singular square matrix of order 3×3 find $|adj\ A|$ if $|A| = 10$.

32. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ find $\left|\left(A^{-1}\right)^{-1}\right|$.

33. If $A = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ find $|AB|$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

34. Find x, y, z and w if $\begin{bmatrix} x - y & 2x + z \\ 2x - y & 3x + w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

35. Construct a 3×3 matrix $A = [a_{ij}]$ whose elements are given by

$$a_{ij} = \begin{cases} 1 + i + j & \text{if } i \geq j \\ \dfrac{|i - 2j|}{2} & \text{if } i < j \end{cases}$$

36. Find A and B if $2A + 3B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

37. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.

38. Express the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix.

39. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ where n is a natural number.

40. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix D such that $CD - AB = O$.

41. Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

42. Prove that the product of the matrices

$$\begin{bmatrix} \cos^2\theta & \cos\theta\ \sin\theta \\ \cos\theta\ \sin\theta & \sin^2\theta \end{bmatrix} \text{ and } \begin{bmatrix} \cos^2\phi & \cos\phi\ \sin\phi \\ \cos\phi\ \sin\phi & \sin^2\phi \end{bmatrix}$$

is the null matrix, when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

43. If $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ show that $A^2 - 12A - I = 0$. Hence find A^{-1}.

44. If $A=\begin{bmatrix}2 & 3\\ 4 & 7\end{bmatrix}$ find $f(A)$ where $f(x) = x^2 - 5x - 2$.

45. If $A = \begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

46. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

47. If $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ -1 & 3\end{bmatrix}$ then show that $(AB)^{-1} = B^{-1}A^{-1}$.

48. Test the consistency of the following system of equations by matrix method :

$$3x - y = 5;\ 6x - 2y = 3$$

49. Using elementary row transformations, find the inverse of the matrix $A=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$, if possible.

50. By using elementary column transformation, find the inverse of $A=\begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$.

51. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A + A' = \mathrm{I}$, then find the general value of α.

Using properties of determinants, prove the following : Q 52 to Q 59.

52. $$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$$

53. $$\begin{vmatrix}x+2 & x+3 & x+2a\\ x+3 & x+4 & x+2b\\ x+4 & x+5 & x+2c\end{vmatrix}=0 \text{ if } a, b, c \text{ are in } A.P.$$

54. $$\begin{vmatrix}\sin\alpha & \cos\alpha & \sin(\alpha+\delta)\\ \sin\beta & \cos\beta & \sin(\beta+\delta)\\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta)\end{vmatrix}=0$$

55. $$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$$

56. $$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

57. $$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

58. $$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$$

59. Show that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

60. (i) If the points (a, b) (a', b') and $(a - a', b - b')$ are collinear. Show that $ab' = a'b$.

(ii) If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ verity that $|AB| = |A||B|$.

61. Given $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the product AB and also find $(AB)^{-1}$.

62. Solve the following equation for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

63. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that,

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

64. Use matrix method to solve the following system of equations : $5x - 7y = 2$, $7x - 5y = 3$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

65. Obtain the inverse of the following matrix using elementary row operations

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

66. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$.

67. Solve the following system of equations by matrix method, where $x \neq 0$, $y \neq 0$, $z \neq 0$

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10, \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10, \quad \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13.$$

68. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

69. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

70. Compute the inverse of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix} \text{ and verify that } A^{-1} A = I_3.$$

71. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$.

72. Using matrix method, solve the following system of linear equations :

$$2x - y = 4,\ 2y + z = 5,\ z + 2x = 7.$$

73. Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \dfrac{A^2 - 3I}{2}$.

74. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary column transformations.

75. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

76. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A \cdot (adj\ A) = (adj\ A) \cdot A = |A|\ I_3$.

77. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$, hence find A^{-1}.

78. Find the matrix X for which

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} . X . \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

79. By using properties of determinants prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3.$$

80. $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz\left(x+y+z\right)^3.$

81. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$

82. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$. Show that $xyz = -1$.

83. If x, y, z are the 10^{th}, 13^{th} and 15^{th} terms of a G.P. find the value of

$$\Delta = \begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}.$$

84. Using the properties of determinants, show that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

85. Using properties of determinants prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

86. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of equations $3x + 4y + 7z = 14$, $2x - y + 3z = 4$, $x + 2y - 3z = 0$.

ANSWERS

1.	$x = 2$, $y = 7$	2.	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
3.	11.	4.	4
5.	$\begin{bmatrix} 9 & -6 \\ 0 & 29 \end{bmatrix}$.	6.	$\begin{bmatrix} 3 & -5 \\ -3 & -1 \end{bmatrix}$.
7.	$AB = [26]$.	8.	$x = 5$
9.	$x = -5$	10.	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
11.	$a^2 + b^2 + c^2 + d^2$.	12.	$x = -13$
13.	$k = \frac{3}{2}$	14.	$\|A\| = 1$.
15.	46	16.	-4

17. $P = -8$

18. $x = \frac{5}{3}$.

19. $k = \frac{10}{3}$.

20. 54.

21. 40.

22. 729

23. 9 sq. units

24. $x = \pm 2$

25. 0

26. 0

27. 9

28. $\begin{bmatrix} 8 & 3 \\ -6 & 5 \end{bmatrix}$.

29. 1728

30. $|A| = 9$

31. 100

32. 11

33. $|AB| = -11$

34. $x = 1,\ y = 2,\ z = 3,\ w = 4$

35. $\begin{bmatrix} 3 & 3/2 & 5/2 \\ 4 & 5 & 2 \\ 5 & 6 & 7 \end{bmatrix}$.

36. $A = \begin{bmatrix} \frac{11}{7} & -\frac{9}{7} & \frac{9}{7} \\ \frac{1}{7} & \frac{18}{7} & \frac{4}{7} \end{bmatrix}$, $B = \begin{bmatrix} -\frac{5}{7} & -\frac{2}{7} & \frac{1}{7} \\ \frac{4}{7} & -\frac{12}{7} & -\frac{5}{7} \end{bmatrix}$

40. $D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$.

41. $x = -2$ or -14

43. $A^{-1} = \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$.

44. $f(A) = 0$

45. $x = 9,\ y = 14$

46. $x = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

48. Inconsistent

49. Inverse does not exist.

50. $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}.$

51. $\alpha = 2n\pi \pm \frac{\pi}{3},\ n \in z$

61. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix},\ (AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}.$

62 0, $3a$

64. $x = \frac{11}{24},\ y = \frac{1}{24}.$

65. $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}.$

66. $x = 0,\ y = 5,\ z = 3$

67. $x = \frac{1}{2},\ y = \frac{1}{3},\ z = \frac{1}{5}$

68. $A^{-1} = -\frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

69. $x = 1,\ y = -2,\ z = 2$

70. $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

71. $(AB)^{-1} = \frac{1}{19}\begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}.$

72. $x = 3,\ y = 2,\ z = 1.$

73. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$

74. $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

75. $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$.

77. $A^{-1} = \dfrac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$.

78. $X = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$.

83. 0

86. $x = 1,\ y = 1,\ z = 1.$

CHAPTER 5

CONTINUITY AND DIFFERENTIATION

POINTS TO REMEMBER

- A function $f(x)$ is said to be continuous at $x = c$ iff $\lim_{x \to c} f(x) = f(c)$

  i.e., $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$

- $f(x)$ is continuous in (a, b) iff it is continuous at $x = c \ \forall \ c \in (a, b)$.

- $f(x)$ is continuous in $[a, b]$ iff

  (i) $f(x)$ is continuous in (a, b)

  (ii) $\lim_{x \to a^+} f(x) = f(a)$,

  (iii) $\lim_{x \to b^-} f(x) = f(b)$

- Trigonometric functions are continuous in their respective domains.

- Every polynomial function is continuous on R.

- If $f(x)$ and $g(x)$ are two continuous functions and $c \in R$ then at $x = a$

  (i) $f(x) \pm g(x)$ are also continuous functions at $x = a$.

  (ii) $g(x) \cdot f(x)$, $f(x) + c$, $cf(x)$, $|f(x)|$ are also continuous at $x = a$.

  (iii) $\dfrac{f(x)}{g(x)}$ is continuous at $x = a$ provided $g(a) \neq 0$.

- $f(x)$ is derivable at $x = c$ in its domain iff

$$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}, \quad \text{and is finite}$$

The value of above limit is denoted by $f'(c)$ and is called the derivative of $f(x)$ at $x = c$.

- $\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

- If $y = f(u)$ and $u = g(t)$ then $\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = f'(u).g'(t)$ (Chain Rule)

- If $y = f(u)$, $x = g(u)$ then,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{f'(u)}{g'(u)}.$$

- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$, $\quad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$, $\quad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1 + x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$, $\quad \frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2 - 1}}$

$\frac{d}{dx}(e^x) = e^x$, $\quad \frac{d}{dx}(\log x) = \frac{1}{x}$

- $f(x) = [x]$ is discontinuous at all integral points and continuous for all $x \in R - Z$.

- **Rolle's theorem :** If $f(x)$ is continuous in $[a, b]$, derivable in (a, b) and $f(a) = f(b)$ then there exists atleast one real number $c \in (a, b)$ such that $f'(c) = 0$.

- **Mean Value Theorem :** If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) then there exists atleast one real number $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
- $f(x) = \log_e x$, $(x > 0)$ is continuous function.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. For what value of x, $f(x) = |2x - 7|$ is not derivable.
2. Write the set of points of continuity of $g(x) = |x - 1| + |x + 1|$.
3. What is derivative of $|x - 3|$ at $x = -1$.
4. What are the points of discontinuity of $f(x) = \dfrac{(x-1)+(x+1)}{(x-7)(x-6)}$.
5. Write the number of points of discontinuity of $f(x) = [x]$ in $[3, 7]$.
6. The function, $f(x) = \begin{cases} \lambda x - 3 & \text{if } x < 2 \\ 4 & \text{if } x = 2 \\ 2x & \text{if } x > 2 \end{cases}$ is a continuous function for all $x \in R$, find λ.
7. For what value of K, $f(x) = \begin{cases} \dfrac{\tan 3x}{\sin 2x}, & x \neq 0 \\ 2K, & x = 0 \end{cases}$ is continuous $\forall\, x \in R$.
8. Write derivative of $\sin x$ *w.r.t.* $\cos x$.
9. If $f(x) = x^2 g(x)$ and $g(1) = 6$, $g'(1) = 3$ find value of $f'(1)$.
10. Write the derivative of the following functions :

    (i) $\log_3 (3x + 5)$ (ii) $e^{\log_2 x}$

    (iii) $e^{6 \log_e (x-1)}, x > 1$

(iv) $\sec^{-1}\sqrt{x} + \operatorname{cosec}^{-1}\sqrt{x}, x \geq 1.$

(v) $\sin^{-1}\left(x^{7/2}\right)$ (vi) $\log_x 5,\ x > 0.$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Discuss the continuity of following functions at the indicated points.

(i) $f(x) = \begin{cases} \dfrac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at $x = 0.$

(ii) $g(x) = \begin{cases} \dfrac{\sin 2x}{3x}, & x \neq 0 \\ \dfrac{3}{2} & x = 0 \end{cases}$ at $x = 0.$

(iii) $f(x) = \begin{cases} x^2 \cos(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0.$

(iv) $f(x) = |x| + |x - 1|$ at $x = 1.$

(v) $f(x) = \begin{cases} x - [x], & x \neq 1 \\ 0 & x = 1 \end{cases}$ at $x = 1.$

12. For what value of k, $f(x) = \begin{cases} 3x^2 - kx + 5, & 0 \leq x < 2 \\ 1 - 3x & 2 \leq x \leq 3 \end{cases}$ is continuous $\forall\ x \in [0, 3].$

13. For what values of a and b

$$f(x) = \begin{cases} \dfrac{x + 2}{|x + 2|} + a & \text{if } x < -2 \\ a + b & \text{if } x = -2 \\ \dfrac{x + 2}{|x + 2|} + 2b & \text{if } x > -2 \end{cases}$$ is continuous at $x = 2.$

14. Prove that $f(x) = |x + 1|$ is continuous at $x = -1$, but not derivable at $x = -1$.

15. For what value of p,

$$f(x) = \begin{cases} x^p \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is derivable at $x = 0$.

16. If $y = \frac{1}{2}\left[\tan^{-1}\left(\frac{2x}{1-x^2}\right) + 2\tan^{-1}\left(\frac{1}{x}\right)\right]$, $0 < x < 1$, find $\frac{dy}{dx}$.

17. If $y = \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ then $\frac{dy}{dx} = ?$

18. If $5^x + 5^y = 5^{x+y}$ then prove that $\frac{dy}{dx} + 5^{y-x} = 0$.

19. If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$ then show that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$.

20. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

21. If $(x + y)^{m+n} = x^m \cdot y^n$ then prove that $\frac{dy}{dx} = \frac{y}{x}$.

22. Find the derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

23. Find the derivative of $\log_e(\sin x)$ w.r.t. $\log_a(\cos x)$.

24. If $x^y + y^x + x^x = m^n$, then find the value of $\frac{dy}{dx}$.

25. If $x = a \cos^3\theta$, $y = a \sin^3\theta$ then find $\frac{d^2y}{dx^2}$.

26. If $x = ae^t$ (sint − cos t)

$y = ae^t$ (sint + cost) then show that $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$ is 1.

27. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $-\dfrac{dy}{dx}$.

28. If $y = x^{\log_e x} + (\log_e x)^x$ then find $\dfrac{dy}{dx}$.

29. Differentiate x^{x^x} w.r.t. x.

30. Find $\dfrac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$

31. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$ where $\dfrac{\pi}{2} < x < \pi$ find $\dfrac{dy}{dx}$.

32. If $x = \sin\left(\dfrac{1}{a}\log_e y\right)$ then show that $(1 - x^2)\, y'' - xy' - a^2y = 0$.

33. Differentiate $(\log x)^{\log x}, x > 1$ $w.r.t.$ x

34. If $\sin y = x \sin(a + y)$ then show that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$.

35. If $y = \sin^{-1}x$, find $\dfrac{d^2y}{dx^2}$ in terms of y.

36. If $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then show that $\dfrac{d^2y}{dx^2} = \dfrac{-b^4}{a^2y^3}$.

37. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$

38. If $y^3 = 3ax^2 - x^3$ then prove that $\dfrac{d^2y}{dx^2} = \dfrac{-2a^2x^2}{y^5}$.

39. Verify Rolle's theorem for the function, $y = x^2 + 2$ in the interval $[a, b]$ where $a = -2$, $b = 2$.

40. Verify Mean Value Theorem for the function, $f(x) = x^2$ in [2, 4]

ANSWERS

1. $x = -7/2$
2. R
3. -1
4. $x = 6, 7$
5. Points of discontinuity of $f(x)$ are 4, 5, 6, 7 *i.e.* four points.

   Note : At $x = 3$, $f(x) = [x]$ is continuous. because $\lim_{x\to 3^+} f(x) = 3 = f(3)$.
6. $\lambda = \frac{7}{2}$.
7. $k = \frac{3}{4}$.
8. $-\cot x$
9. 15
10. (i) $\frac{3}{3x + 5} \log_3 e$ (ii) $e^{\log_2 x} \frac{1}{x}.\log_2 e$.

    (iii) $6\,(x - 1)^5$ (iv) 0

    (v) $\frac{7}{2}\frac{x^2\sqrt{x}}{\sqrt{1-x^7}}$. (vi) $\frac{-\log_e 5}{x\left(\log_e x\right)^2}$.
11. (i) Discontinuous (ii) Discontinuous

    (iii) Continuous (iv) continuous

    (v) Discontinuous
12. k = 11
13. $a = 0$, $b = -1$.
15. $p > 1$.
16. 0
17. $\frac{-x}{\sqrt{1-x^2}}$.
22. 1
23. $-\cot^2 x \ \log_e a$
24. $\frac{dy}{dx} = \frac{-x^x\left(1+\log x\right) - yx^{y-1} - y^x \log y}{x^y \log x + xy^{x-1}}$.

25. $\dfrac{d^2y}{dx^2} = \dfrac{1}{3a}\operatorname{cosec}\theta \sec^4\theta.$

27. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x}\sqrt{1-x}}.$

28. $x^{\log x}\dfrac{2\log x}{x} + (\log x)^x\left[\dfrac{1}{\log x} + \log(\log x)\right].$

29. $\dfrac{dy}{dx} = x^{x^x}.\, x^x \log x\left(1 + \log x + \dfrac{1}{x\log x}\right).$

30. $\dfrac{dy}{dx} = \dfrac{y\tan x + \log\cos y}{x\tan y + \log\cos x}$

31. $\dfrac{dy}{dx} = -\dfrac{1}{2}.$

Hint. : $\sin\dfrac{x}{2} > \cos\dfrac{x}{2}$ *for* $x \in \left(\dfrac{\pi}{2}, \pi\right).$

33. $(\log x)^{\log x}\left[\dfrac{1}{x} + \dfrac{\log(\log x)}{x}\right],\ x > 1$

35. $\sec^2 y\ \tan y.$

CHAPTER 6

APPLICATIONS OF DERIVATIVES

POINTS TO REMEMBER

- **Rate of Change :** Let $y = f(x)$ be a function then the rate of change of y with respect to x is given by $\frac{dy}{dx} = f'(x)$ where a quantity y varies with another quantity x.

  $\left.\frac{dy}{dx}\right]_{x=x_0}$ or $f'(x_0)$ represents the rate of change of y w.r.t. x at $x = x_0$.

- If $x = f(t)$ and $y = g(t)$

  By chain rule

  $$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} \text{ if } \frac{dx}{dt} \neq 0.$$

- (i) A function $f(x)$ is said to be increasing (non-decreasing) on an interval (a, b) if $x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \leq f(x_2) \ \forall \ x_1, x_2 \in (a, b)$. Alternatively if $f'(x_0) \geq 0 \ \forall \ x \in (a, b)$, then $f(x)$ is increasing function in (a, b).

  (ii) A function $f(x)$ is said to be decreasing (non-increasing) on an interval (a, b). If $x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \geq f(x_2) \ \forall \ x_1, x_2 \in (a, b)$. Alternatively if $f'(x) \leq 0 \ \forall \ x \in (a, b)$, then $f(x)$ is decreasing function in (a, b).

- The equation of tangent at the point (x_0, y_0) to a curve $y = f(x)$ is given by

  $$y - y_0 = \left.\frac{dy}{dx}\right]_{(x_0, y_0)} (x - x_0).$$

where $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ = slope of the tangent at the point (x_0, y_0).

(i) If $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ does not exist then tangent is parallel to y-axis at (x_0, y_0) and its equation is $x = x_0$.

(ii) If tangent at $x = x_0$ is parallel to x-axis then $\left.\frac{dy}{dx}\right]_{x=x_0} = 0$

- Slope of the normal to the curve at the point (x_0, y_0) is given by $-\dfrac{1}{\left.\frac{dy}{dx}\right]_{x=x_0}}$.

- Equation of the normal to the curve $y = f(x)$ at a point (x_0, y_0) is given by

$$y - y_0 = -\frac{1}{\left.\frac{dy}{dx}\right]_{(x_0, y_0)}}(x - x_0).$$

- If $\left.\frac{dy}{dx}\right]_{(x_0, y_0)} = 0.$ then equation of the normal is $x = x_0$.

- If $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ does not exit, then the normal is parallel to x-axis and the equation of the normal is $y = y_0$.

- Let $y = f(x)$

  Δx = the small increment in x and

  Δy be the increment in y corresponding to the increment in x

  Then approximate change in y is given by

  $$dy = \left(\frac{dy}{dx}\right)\Delta x \qquad \text{or} \qquad dy = f'(x)\,\Delta x$$

  The approximate change in the value of f is given by

  $$f(x + \Delta x) = f(x) + f'(x)\,\Delta x$$

- Let f be a function. Let point c be in the domain of the function f at which either $f'(x) = 0$ or f is not derivable is called a critical point of f.

- **First Derivative Test :** Let f be a function defined on an open interval I. Let f be continuous at a critical point $c \in$ I. Then if,

  (i) $f'(x)$ changes sign from positive to negative as x increases through c, then c is called the point of the local maxima.

  (ii) $f'(x)$ changes sign from negative to positive as x increases through c, then c is a point of *local minima*.

  (iii) $f'(x)$ does not change sign as x increases through c, then c is neither a point of *local maxima* nor a point of *local minima*. Such a point is called a point of *inflexion.*

- **Second Derivative Test :** Let f be a function defined on an interval I and let $c \in$ I. Then

  (i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$.

  $f(c)$ is local maximum value of f.

  (ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$. $f(c)$ is local minimum value of f.

  (iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of perimeter of the square.

2. The radius of the circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

3. If the radius of a soap bubble is increasing at the rate of $\frac{1}{2}\text{cm/sec}$. At what rate its volume is increasing when the radius is 1 cm.

4. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/sec. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

5. The total revenue in rupees received from the sale of x units of a product is given by

   $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

6. Find the maximum and minimum values of function $f(x) = \sin 2x + 5$.

7. Find the maximum and minimum values (if any) of the function

   $$f(x) = -|x-1| + 7 \quad \forall\, x \in R.$$

8. Find the value of a for which the function $f(x) = x^2 - 2ax + 6$, $x > 0$ is strictly increasing.

9. Write the interval for which the function $f(x) = \cos x$, $0 \leq x \leq 2\pi$ is decreasing.

10. What is the interval on which the function $f(x) = \dfrac{\log x}{x}, x \in (0, \infty)$ is increasing?

11. For which values of x, the functions $y = x^4 - \dfrac{4}{3}x^3$ is increasing?

12. Write the interval for which the function $f(x) = \dfrac{1}{x}$ is strictly decreasing.

13. Find the sub-interval of the interval $(0, \pi/2)$ in which the function $f(x) = \sin 3x$ is increasing.

14. Without using derivatives, find the maximum and minimum value of $y = |3 \sin x + 1|$.

15. If $f(x) = ax + \cos x$ is strictly increasing on R, find a.

16. Write the interval in which the function $f(x) = x^9 + 3x^7 + 64$ is increasing.

17. What is the slope of the tangent to the curve $f = x^3 - 5x + 3$ at the point whose x co-ordinate is 2?

18. At what point on the curve $y = x^2$ does the tangent make an angle of 45° with positive direction of the x-axis?

19. Find the point on the curve $y = 3x^2 - 12x + 9$ at which the tangent is parallel to x-axis.

20. What is the slope of the normal to the curve $y = 5x^2 - 4 \sin x$ at $x = 0$.

21. Find the point on the curve $y = 3x^2 + 4$ at which the tangent is perpendicular to the line with slope $-\frac{1}{6}$.

22. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the y – co-ordinate.

23. If the curves $y = 2e^x$ and $y = ae^{-x}$ intersect orthogonally (cut at right angles), what is the value of a?

24. Find the slope of the normal to the curve $y = 8x^2 - 3$ at $x = \frac{1}{4}$.

25. Find the rate of change of the total surface area of a cylinder of radius r and height h with respect to radius when height is equal to the radius of the base of cylinder.

26. Find the rate of change of the area of a circle with respect to its radius. How fast is the area changing w.r.t. its radius when its radius is 3 cm?

27. For the curve $y = (2x + 1)^3$ find the rate of change of slope at $x = 1$.

28. Find the slope of the normal to the curve

$$x = 1 - a \sin \theta \quad ; \quad y = b \cos^2\theta \quad \text{at} \quad \theta = \frac{\pi}{2}$$

29. If a manufacturer's total cost function is $C(x) = 1000 + 40x + x^2$, where x is the out put, find the marginal cost for producing 20 units.

30. Find 'a' for which $f(x) = a(x + \sin x)$ is strictly increasing on R.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

31. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y co-ordinate is changing 8 times as fast as the x co-ordinate.

32. A ladder 5 metres long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 metres away from the wall?

33. A balloon which always remain spherical is being inflated by pumping in 900 cubic cm of a gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

34. A man 2 meters high walks at a uniform speed of 5 km/hr away from a lamp post 6 metres high. Find the rate at which the length of his shadow increases.

35. Water is running out of a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and altitude is 20 cm, find the rate at which the water level is dropping when it is 5 cm from the top.

36. The length x of a rectangle is decreasing at the rate of 5 cm/sec and the width y is increasing as the rate of 4 cm/sec when $x = 8$ cm and $y = 6$ cm. Find the rate of change of

    (a) Perimeter (b) Area of the rectangle.

37. Sand is pouring from a pipe at the rate of 12c.c/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when height is 4 cm?

38. The area of an expanding rectangle is increasing at the rate of 48 cm^2/sec. The length of the rectangle is always equal to the square of the breadth. At what rate is the length increasing at the instant when the breadth is 4.5 cm?

39. Find a point on the curve $y = (x - 3)^2$ where the tangent is parallel to the line joining the points (4, 1) and (3, 0).

40. Find the equation of all lines having slope zero which are tangents to the curve $y = \dfrac{1}{x^2 - 2x + 3}$.

41. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

42. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

43. Show that the curves $4x = y^2$ and $4xy = k$ cut as right angles if $k^2 = 512$.

44. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - y + 5 = 0$.

45. Find the equation of the tangent to the curve $\sqrt{x}+\sqrt{y}=a$ at the point $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$.

46. Find the points on the curve $4y = x^3$ where slope of the tangent is $\frac{16}{3}$.

47. Show that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the y-axis.

48. Find the equation of the tangent to the curve given by $x = a \sin^3 t$, $y = b \cos^3 t$ at a point where $t=\frac{\pi}{2}$.

49. Find the intervals in which the function $f(x) = \log(1 + x) - \frac{x}{1+x}, x > -1$ is increasing or decreasing.

50. Find the intervals in which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is

    (a) Increasing (b) Decreasing.

51. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in [0, 1].

52. Find the intervals on which the function $f(x)=\frac{x}{x^2+1}$ is decreasing.

53. Prove that $f(x)=\frac{x^3}{3}-x^2+9x,\ x \in [1, 2]$ is strictly increasing. Hence find the minimum value of $f(x)$.

54. Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x,\ 0 \le x \le \frac{\pi}{2}$ is increasing or decreasing.

55. Find the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

56. Find the interval in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}},\ x > 0$ is strictly decreasing.

57. Show that the function $f(x) = \tan^{-1} (\sin x + \cos x)$, is strictly increasing on the interval $\left(0, \frac{\pi}{4}\right)$.

58. Show that the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is strictly increasing on $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$.

59. Show that the function $f(x) = \frac{\sin x}{x}$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

Using differentials, find the approximate value of (Q. No. 60 to 64).

60. $(0.009)^{\frac{1}{3}}$.

61. $(255)^{\frac{1}{4}}$.

62. $(0.0037)^{\frac{1}{2}}$.

63. $\sqrt{0.037}$.

64. $\sqrt{25.3}$.

65. Find the approximate value of f (5.001) where $f(x) = x^3 - 7x^2 + 15$.

66. Find the approximate value of f (3.02) where $f(x) = 3x^2 + 5x + 3$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

67. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.

68. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is maximum.

69. Show that of all the rectangles of given area, the square has the smallest perimeter.

70. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radium of the base.

71. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

72. A point on the hypotenuse of a triangle is at a distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

73. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

74. Find the interval in which the function f given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

75. Find the intervals in which the function $f(x) = (x + 1)^3 (x - 3)^3$ is strictly increasing or strictly decreasing.

76. Find the local maximum and local minimum of $f(x) = \sin 2x - x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

77. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to x-axis.

78. A solid is formed by a cylinder of radius r and height h together with two hemisphere of radius r attached at each end. It the volume of the solid is constant but radius r is increasing at the rate of $\frac{1}{2\pi}$ metre/min. How fast must h (height) be changing when r and h are 10 metres.

79. Find the equation of the normal to the curve

$x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$ at the point θ and show that its distance from the origin is a.

80. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

81. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

82. Find the equation of the tangents at the points where the curve $2y = 3x^2 - 2x - 8$ cuts the x-axis and show that they make supplementary angles with the x-axis.

83. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

84. A window is in the form of a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16 metres. Find the width of the window in order that the maximum amount of light may be admitted.

85. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet?

86. Find a point on the parabola $y^2 = 4x$ which is nearest to the point (2, –8).

87. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each cover and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum.

88. A window in the form of a rectangle is surmounted by a semi circular opening. The total perimeter of the window is 30 metres. Find the dimensions of the rectangular part of the window to admit maximum light through the whole opening.

89. An open box with square base is to be made out of a given iron sheet of area 27 sq. meter, show that the maximum value of the box is 13.5 cubic metres.

90. A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into a square and other in to a circle. What should be the length of two pieces so that the combined area of the square and the circle is minimum?

91. Show that the height of the cylinder of maximum volume which can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

92. Show that the altitude of the right circular cone of maximum volume that can be inscribed is a sphere of radius r is $\frac{4r}{3}$.

93. Prove that the surface area of solid cuboid of a square base and given volume is minimum, when it is a cube.

94. Show that the volume of the greatest cylinder which can be inscribed in a right circular cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

95. Show that the right triangle of maximum area that can be inscribed in a circle is an isosceles triangle.

96. A given quantity of metal is to be cast half cylinder with a rectangular box and semicircular ends. Show that the total surface area is minimum when the ratio of the length of cylinder to the diameter of its semicircular ends is $\pi : (\pi + 2)$.

ANSWERS

1. 0.8 cm/sec.
2. 4.4 cm/sec.
3. 2π cm^3/sec.
4. 80π cm^2/sec.
5. Rs. 208.
6. Minimum value = 4, maximum value = 6.
7. Maximum value = 7, minimum value does not exist.
8. $a \le 0$.
9. $[0, \pi]$
10. $(0, e]$
11. $x \ge 1$
12. $(-\infty, 0) \cup (0, \infty)$
13. $\left(0, \frac{\pi}{6}\right)$.
14. Maximum value = 4, minimum valve = 0.
15. $a > 1$.
16. R
17. 7
18. $\left(\frac{1}{2}, \frac{1}{4}\right)$.
19. $(2, -3)$
20. $\frac{1}{4}$
21. $(1, 7)$

22. (0, 0), (2, 4)

23. $\frac{1}{2}$.

24. $-\frac{1}{4}$.

25. $6\pi r$

26. $2\ \pi\ cm^2/cm$

27. 72

28. $-\frac{a}{2b}$.

29. Rs. 80.

30. $a > 0$.

31. $(4, 11)$ and $\left(-4, -\frac{31}{3}\right)$.

32. $-\frac{8}{3}$ cm/sec.

33. $\frac{1}{\pi}$ cm/sec.

34. 2.5 km/hr.

35. $\frac{4}{45\pi}$ cm/sec.

36. (a) –2 cm/min, (b) 2 cm^2/min

37. $\frac{1}{48\pi}$ cm/sec.

38. 7.11 cm/sec.

39. $\left(\frac{7}{2}, \frac{1}{4}\right)$.

40. $y = \frac{1}{2}$.

42. $2x + 3my = am^2\ (2 + 3m^2)$

44. $48x - 24y = 23$

45. $2x + 2y = a^2$

46. $\left(\frac{8}{3}, \frac{128}{27}\right), \left(\frac{-8}{3}, -\frac{128}{27}\right)$.

48. $y = 0$

49. Increasing in $(0, \infty)$, decreasing in $(-1, 0)$.

50. Increasing in $(-\infty, 2) \cup (6, \infty)$, Decreasing in $(2, 6)$.

52. (– ∞, –1) and (1, ∞).

53. $\dfrac{25}{3}$.

54. Increasing in $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ Decreasing in $\left(0, \dfrac{\pi}{4}\right)$.

55. $a = -2$.

56. Strictly decreasing in (1, ∞).

60. 0.2083

61. 3.9961

62. 0.06083

63. 0.1925

64. 5.03

65. –34.995

66. 45.46

68. 25, 10

74. Strictly increasing in $\left[0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{5\pi}{4}, 2\pi\right]$

Strictly decreasing in $\left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right)$.

75. Strictly increasing in $(1, 3) \cup (3, \infty)$

Strictly decreasing in $(-\infty, -1) \cup (-1, 1)$.

76. Local maxima at $x = \dfrac{\pi}{6}$

Local max. value $= \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}$

Local minima at $x = -\dfrac{\pi}{6}$

Local minimum value $= \dfrac{-\sqrt{3}}{2} + \dfrac{\pi}{6}$

77. Strictly increasing in $(-\infty, 2] \cup [3, \infty)$

Strictly decreasing in (2, 3).

Points are (2, 29) and (3, 28).

78. $-\frac{3}{\pi}$metres/min.

79. $x + y \tan\theta - a \sec\theta = 0$.

80. (0, 0), (–1, –2) and (1, 2).

81. $x + y = 3$

82. $5x - y - 10 = 0$ and $15x + 3y + 20 = 0$

83. $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1, \quad \frac{y - y_0}{a^2 y_0} + \frac{x - x_0}{b^2 x_0} = 0.$

84. $\frac{16}{6 - \sqrt{3}}$

85. $\sqrt{5}$

86. (4, –4)

87. 3cm

88. $\frac{60}{\pi + 4}, \frac{30}{\pi + 4}.$

90. $\frac{112}{\pi + 4}$cm, $\frac{28\pi}{\pi + 4}$cm.

CHAPTER 7

INTEGRALS

POINTS TO REMEMBER

- Integration is the reverse process of Differentiation.
- Let $\frac{d}{dx}F(x)=f(x)$ then we write $\int f(x)dx=F(x)+c$.
- These integrals are called indefinite integrals and c is called constant of integration.
- From geometrical point of view an indefinite integral is collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y-axis.

STANDARD FORMULAE

1. $$\int x^n dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log|x| + c & n = -1 \end{cases}$$

2. $$\int (ax+b)^n\, dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{(n+1)\,a} + c & n \neq -1 \\ \dfrac{1}{a}\log|ax+b| + c & n = -1 \end{cases}$$

3. $\int \sin x\ dx = -\cos x + c.$ 4. $\int \cos x\ dx = \sin x + c.$

5. $\int \tan x \,.\, dx = -\log|\cos x| + c = \log|\sec x| + c.$

6. $\int \cot x \, dx = \log|\sin x| + c.$ 7. $\int \sec^2 x \, . dx = \tan x + c.$

8. $\int \text{cosec}^2 x \, . dx = -\cot x + c.$ 9. $\int \sec x \, . \tan x \, . dx = \sec x + c.$

10. $\int \text{cosec } x \cot x \, dx = -\text{ cosec } x + c.$

11. $\int \sec x \, dx = \log|\sec x + \tan x| + c.$

12. $\int \text{cosec } x \, dx = \log|\text{cosec } x - \cot x| + c.$

13. $\int e^x dx = e^x + c.$ 14. $\int a^x dx = \dfrac{a^x}{\log a} + c$

15. $\int \dfrac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c, |x < 1|.$

16. $\int \dfrac{1}{1 + x^2} dx = \tan^{-1} x + c.$

17. $\int \dfrac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c, |x| > 1.$

18. $\int \dfrac{1}{a^2 - x^2} dx = \dfrac{1}{2a} \log\left|\dfrac{a + x}{a - x}\right| + c.$

19. $\int \dfrac{1}{x^2 - a^2} dx = \dfrac{1}{2a} \log\left|\dfrac{x - a}{x + a}\right| + c.$

20. $\int \dfrac{1}{a^2 + x^2} dx = \dfrac{1}{a} \tan^{-1} \dfrac{x}{a} + c.$

21. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c.$

22. $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c.$

23. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

24. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c.$

25. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \log\left|x + \sqrt{a^2 + x^2}\right| + c.$

26. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

RULES OF INTEGRATION

1. $\int k.f(x)dx = k\int f(x)dx.$

2. $\int k\{f(x) \pm g(x)\}dx = k\int f(x)dx \pm k\int g(x)dx.$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c.$

2. $\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c.$

3. $\displaystyle\int \frac{f'(x)}{[f(x)]^n}\,dx = \frac{(f(x))^{-n+1}}{-n+1} + c.$

INTEGRATION BY PARTS

$$\int f(x).\,g(x)\,dx = f(x).\left[\int g(x)\,dx\right] - \int f'(x).\left[\int g(x)\,dx\right]dx.$$

DEFINITE INTEGRALS

$$\int_a^b f(x)\,dx = F(b) - F(a),\ \text{where } F(x) = \int f(x)\,dx.$$

DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[\begin{array}{l} f(a) + f(a+h) + f(a+2h) \\ \qquad + \ldots.. + f\left(a + \overline{n-1}\,h\right)\end{array}\right]$$

where $h = \dfrac{b-a}{h}$. or $\displaystyle\int_a^b f(x)\,dx = \lim_{h\to 0}\left[h\sum_{r=1}^{n} f(a+rh)\right]$

PROPERTIES OF DEFINITE INTEGRAL

1. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

2. $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(t)\,dt.$

3. $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$

4. (i) $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx.$ (ii) $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)\,dx.$

5. $\int_{-a}^{a} f(x) = 0$; if $f(x)$ is odd function.

6. $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, if $f(x)$ is even function.

7. $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x)=f(x) \\ 0, & \text{if } f(2a-x)=-f(x) \end{cases}$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following integrals

1. $\int \left(\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\right)dx.$

2. $\int_{-1}^{1} e^{|x|}dx.$

3. $\int \frac{1}{1-\sin^2 x}dx.$

4. $\int \left(8^x + x^8 + \frac{8}{x} + \frac{x}{8}\right)dx.$

5. $\int_{-1}^{1} x^{99}\cos^4 x\,dx.$

6. $\int \frac{1}{x\log x\log(\log x)}dx.$

7. $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx.$

8. $\int \left(e^{a\log x} + e^{x\log a}\right)dx.$

9. $\int \left(\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\right)dx.$

10. $\int_{-\frac{\pi}{2}}^{\pi/2} \sin^7 x\,dx.$

11. $\int \left(x^c + c^x\right)dx.$

12. $\frac{d}{dx}\left[\int f(x)dx\right].$

13. $\int \frac{1}{\sin^2 x \cos^2 x} dx.$

14. $\int \frac{1}{\sqrt{x} + \sqrt{x-1}} dx.$

15. $\int e^{-\log e^x} dx.$

16. $\int \frac{e^x}{a^x} dx.$

17. $\int 2^x e^x dx.$

18. $\int \frac{x}{\sqrt{x+1}} dx.$

19. $\int \frac{x}{(x+1)^2} dx.$

20. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$

21. $\int \cos^2 \alpha \, dx.$

22. $\int \frac{1}{x \cos \alpha + 1} dx.$

23. $\int \sec x . \log(\sec x + \tan x) \, dx.$

24. $\int \frac{1}{\cos \alpha + x \sin \alpha} dx.$

25. $\int \cot x . \log \sin x \, dx.$

26. $\int \left(x - \frac{1}{2} \right)^3 dx.$

27. $\int \frac{1}{x(2 + 3 \log x)} dx.$

28. $\int \frac{1 - \sin x}{x + \cos x} dx.$

29. $\int \frac{1 - \cos x}{\sin x} dx.$

30. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx.$

31. $\int \frac{(x+1)}{x} (x + \log x) dx.$

32. $\int \left(\sqrt{ax} - \frac{1}{\sqrt{ax}} \right)^2 dx.$

33. $\int_0^{\pi} |\cos x| dx.$

34. $\int_0^2 [x] dx$ where [ ] is greatest integer function.

35. $\int_0^{\sqrt{2}} [x^2] dx$ where [ ] is greatest integer function.

36. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx.$ 37. $\int_{-2}^{1} \frac{|x|}{x} dx.$

38. $\int_{-1}^{1} x|x| dx.$

39. If $\int_0^a \frac{1}{1+x^2} = \frac{\pi}{4}$, then what is value of a.

40. $\int_a^b f(x) dx + \int_b^a f(x) dx.$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

41. (i) $\int \frac{x \text{ cosec} \left(\tan^{-1} x^2\right)}{1 + x^4} dx.$ (ii) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx.$

(iii) $\int \frac{1}{\sin(x-a)\sin(x-b)} dx.$ (iv) $\int \frac{\cos(x+a)}{\cos(x-a)} dx.$

(v) $\int \cos x \cos 2x \cos 3x \; dx.$ (vi) $\int \cos^5 x \; dx.$

(vii) $\int \sin^2 x \cos^4 x \; dx.$ (viii) $\int \cot^3 x \text{ cosec}^4 x \; dx.$

(ix) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx.$ (x) $\int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}} dx.$

(xi) $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx.$ (xii) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx.$

42. Evaluate :

(i) $\int \frac{x}{x^4 + x^2 + 1} dx.$

*(ii) $\int \frac{1}{x\left[6(\log x)^2 + 7\log x + 2\right]} dx.$

(iii) $\int \frac{dx}{1 + x - x^2}.$

(iv) $\int \frac{1}{\sqrt{9 + 8x - x^2}} dx.$

(v) $\int \frac{1}{\sqrt{(x - a)(x - b)}} dx.$

(vi) $\int \sqrt{\frac{\sin(x - \alpha)}{\sin(x + \alpha)}}\, dx.$

(vii) $\int \frac{5x - 2}{3x^2 + 2x + 1} dx.$

(viii) $\int \frac{x^2}{x^2 + 6x + 12} dx.$

(ix) $\int \frac{x + 2}{\sqrt{4x - x^2}} dx.$

(x) $\int x\sqrt{1 + x - x^2}\, dx.$

(xi) $\int (3x - 2)\sqrt{x^2 + x + 1}\, dx.$

(xii) $\int \sqrt{\sec x + 1}\, dx.$

43. Evaluate :

(i) $\int \frac{dx}{x(x^7 + 1)}.$

(ii) $\int \frac{\sin x}{(1 + \cos x)(2 + 3\cos x)} dx.$

(iii) $\int \frac{\sin\theta\cos\theta}{\cos^2\theta - \cos\theta - 2} d\theta.$

(iv) $\displaystyle\int \frac{x-1}{(x+1)(x-2)(x+3)}\,dx.$

(v) $\displaystyle\int \frac{x^2+x+2}{(x-2)(x-1)}\,dx.$

(vi) $\displaystyle\int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^3+3\right)\left(x^2+4\right)}\,dx.$

(vii) $\displaystyle\int \frac{dx}{(2x+1)\left(x^2+4\right)}.$

(viii) $\displaystyle\int \frac{dx}{\sin x\,(1-2\cos x)}.$

(ix) $\displaystyle\int \frac{\sin x}{\sin 4x}\,dx.$

(x) $\displaystyle\int \frac{x^2-1}{x^4+x^2+1}\,dx.$

(xi) $\displaystyle\int \sqrt{\tan x}\;dx.$

(xii) $\displaystyle\int \frac{x^2+9}{x^4+81}\,dx.$

44. Evaluate :

(i) $\displaystyle\int x^5 \sin x^3 dx.$

(ii) $\displaystyle\int \sec^3 x\,dx.$

(iii) $\displaystyle\int e^{ax}\cos(bx+c)\,dx.$

(iv) $\displaystyle\int \sin^{-1}\frac{6x}{1+9x^2}\,dx.$

(v) $\displaystyle\int \cos\sqrt{x}\,dx.$

(vi) $\displaystyle\int x^3\tan^{-1}x\;dx.$

(vii) $\displaystyle\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx.$

(viii) $\displaystyle\int e^{x}\left(\frac{x-1}{2x^2}\right)dx.$

(ix) $\displaystyle\int \sqrt{2ax-x^2}\,dx.$

(x) $\displaystyle\int e^{x}\frac{\left(x^2+1\right)}{(x+1)^2}\,dx.$

(xi) $\displaystyle\int e^{x}\frac{(2+\sin 2x)}{(1+\cos 2x)}\,dx.$

(xii) $\displaystyle\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx.$

(xiii) $\int (6x + 5)\sqrt{6 + x - x^2}\,dx.$

(xiv) $\int (x - 2)\sqrt{\dfrac{x + 3}{x - 3}}\,dx.$

(xv) $\int (2x - 5)\sqrt{x^2 - 4x + 3}\,dx.$

(xvi) $\int \sqrt{x^2 - 4x + 8}\,dx.$

45. Evaluate the following definite integrals :

(i) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx.$

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos 2x \log \sin x\,dx.$

(iii) $\displaystyle\int_0^1 x\sqrt{\frac{1 - x^2}{1 + x^2}}\,dx.$

(iv) $\displaystyle\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{\left(1 - x^2\right)^{3/2}}\,dx.$

(v) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\,dx.$

(vi) $\displaystyle\int_1^2 \frac{5x^2}{x^2 + 4x + 3}\,dx.$

(vii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x}\,dx.$

46. Evaluate :

(i) $\displaystyle\int_1^3 \{|x - 1| + |x - 2| + |x - 3|\}\,dx.$

(ii) $\displaystyle\int_0^{\pi} \frac{x}{1 + \sin x}\,dx.$

(iii) $\displaystyle\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx.$

(iv) $\displaystyle\int_0^{\frac{\pi}{2}} \log \sin x \; dx.$

(v) $\displaystyle\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx.$

(vi) $\displaystyle\int_{-2}^{2} f(x)\, dx$ where $f(x) = \begin{cases} 2x - x^3 & \text{when } -2 \le x < 1 \\ x^3 - 3x + 2 & \text{when } -1 \le x < 1 \\ 3x - 2 & \text{when } 1 \le x < 2. \end{cases}$

(vii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx.$

(viii) $\displaystyle\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}\, dx.$

47. Evaluate the following integrals

(i) $\displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

(ii) $\displaystyle\int_0^{1} \sin^{-1}\left(\frac{2x}{1 + x^2}\right) dx.$

(iii) $\displaystyle\int_{-1}^{1} \log\left(\frac{1 + \sin x}{1 - \sin x}\right) dx.$

(iv) $\displaystyle\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx.$

(v) $\displaystyle\int_0^{\pi} \frac{x \tan x}{\sec x \;\operatorname{cosec} x}\, dx.$

(iv) $\displaystyle\int_{-a}^{a} \sqrt{\frac{a - x}{a + x}}\, dx.$

48. $\int_0^1 [2x]dx$ where [ ] is greatest integer function.

49. $\int e^{\log x + \log \sin x} dx.$

50. $\int e^{\log(x+1) - \log x} dx.$

51. $\int \frac{\sin x}{\sin 2x} dx.$

52. $\int \sin x \sin 2x \, dx.$

53. $\int_{-\frac{\pi}{4}}^{\pi/4} |\sin x| dx.$

54. $\int_a^b f(x)dx + \int_b^a f(a+b-x)\, dx.$

55. $\int \frac{1}{\sec x + \tan x} dx.$

56. $\int \frac{\sin^2 x}{1 + \cos x} dx.$

57. $\int \frac{1 - \tan x}{1 + \tan x} dx.$

58. $\int \frac{a^x + b^x}{c^x} dx.$

59. Evaluate

(i) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx, \; x \in [0, 1]$

(ii) $\int \sqrt{\frac{1 - \sqrt{x}}{1 + \sqrt{x}}} \, dx$

(iii) $\int \frac{\sqrt{x^2 + 1}\left[\log\left(x^2 + 1\right) - 2\log x\right]}{x^4} dx$

(iv) $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$

(v) $\int \sin^{-1}\sqrt{\frac{x}{a + x}} \, dx$

(vi) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

(vii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin|x| - \cos|x|)\, dx$

(viii) $\int_{1}^{2} [x^2]\, dx$, where $[x]$ is greatest integer function

(ix) $\int_{-1}^{\frac{3}{2}} |x \sin \pi x|\, dx.$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

60. Evaluate the following integrals :

(i) $\int \frac{x^5 + 4}{x^5 - x}\, dx.$

(ii) $\int \frac{dx}{(x-1)(x^2+4)}\, dx$

(iii) $\int \frac{2x^3}{(x+1)(x-3)^2}\, dx$

(iv) $\int \frac{x^4}{x^4 - 16}\, dx$

(v) $\int_{0}^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx.$

(vi) $\int \frac{1}{x^4 + 1}\, dx.$

(vii) $\int_{0}^{\infty} \frac{x \tan^{-1} x}{(1 + x^2)^2}\, dx.$

61. Evaluate the following integrals as limit of sums :

(i) $\int_{2}^{4} (2x + 1)\, dx.$

(ii) $\int_{0}^{2} (x^2 + 3)\, dx.$

(iii) $\int_1^3 \left(3x^2 - 2x + 4\right) dx.$ (iv) $\int_0^4 \left(3x^2 + e^{2x}\right) dx.$

(v) $\int_2^5 \left(x^2 + 3x\right) dx.$

62. Evaluate

(i) $\int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx$

(ii) $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

(iii) $\int_0^1 \frac{\log(1 + x)}{1 + x^2} dx$ (iv) $\int_0^{\frac{\pi}{2}} \left(2\log\sin x - \log\sin 2x\right) dx.$

63. $\int \frac{1}{\sin x + \sin 2x} dx.$ 64. $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta.$

65. $\int \sec^3 x \, dx.$ 66. $\int e^{2x} \cos 3x \, dx.$

ANSWERS

1. $\frac{\pi}{2} x + c.$
2. $2e - 2$
3. $\tan x + c.$
4. $\frac{8^x}{\log 8} + \frac{x^9}{9} + 8\log|x| + \frac{x^2}{16} + c.$
5. 0
6. $\log | \log (\log x) | + c$

7. 0

8. $\dfrac{x^{a+1}}{a+1}+\dfrac{a^x}{\log a}+c$

9. tan x + c

10. 0

11. $\dfrac{x^{c+1}}{c+1}+\dfrac{c^x}{\log c}+c$

12. $f(x)$ + c

13. tan x – cot x + c

14. $\dfrac{2}{3}x^{3/2}-\dfrac{2}{3}(x-1)^{3/2}+c$

15. log $|x|$ + c

16. $\left(\dfrac{e}{a}\right)^x \Big/ \log(e/a)+c$

17. $\dfrac{2^x e^x}{\log(2e)}+c$

18. $\dfrac{2}{3}(x+1)^{3/2}-2(x+1)^{1/2}+c.$

19. $\log|x+1|+\dfrac{1}{x+1}+c.$

20. $2e^{\sqrt{x}}+c$

21. $x\cos^2\alpha + c$

22. $\dfrac{\log|x\cos\alpha+1|}{\cos\alpha}+c.$

23. $\dfrac{(\log|\sec x+\tan x|)^2}{2}+c$

24. $\dfrac{\log|\cos\alpha+x\sin\alpha|}{\sin\alpha}+c$

25. $\dfrac{(\log\sin x)^2}{2}+c$

26. $\dfrac{x^4}{4}+\dfrac{1}{2x^2}-\dfrac{3x^2}{2}+3|\log x|+c.$

27. $\dfrac{1}{3}\log|2+3\log x|+c.$

28. log $|x$ + cos $x|$ c

29. 2 log $|\sec x/2|$ + c.

30. $\dfrac{1}{e}\log|x^e+e^x|+c.$

31. $\dfrac{(x+\log x)^2}{2}+c$

32. $a\dfrac{x^2}{2}+\dfrac{\log|ax|}{a}-2x+c.$

33. 0

34. 1

35. $(\sqrt{2}-1)$

36. $\dfrac{b-a}{2}$

37. –1

38. 0

39. 1

40. 0

41. (i) $\dfrac{1}{2}\log\left[\operatorname{cosec}(\tan^{-1}x^2)-\dfrac{1}{x^2}\right]+c.$

(ii) $\dfrac{1}{2}\left(x^2-x\sqrt{x^2-1}\right)+\dfrac{1}{2}\log\left|x+\sqrt{x^2-1}\right|+c.$

(iii) $\dfrac{1}{\sin(a-b)}\log\left|\dfrac{\sin(x-a)}{\sin(x-b)}\right|+c$

(iv) $x\cos 2a-\sin 2a\log|\sec(x-a)|+c.$

(v) $\dfrac{1}{48}[12x+6\sin 2x+3\sin 4x+2\sin 6x]+c.$

(vi) $\sin x-\dfrac{2}{3}\sin^3 x+\dfrac{1}{5}\sin^5 x+c.$

(vii) $\dfrac{1}{32}\left[2x+\dfrac{1}{2}\sin 2x-\dfrac{1}{2}\sin 4x-\dfrac{1}{6}\sin 6x\right]+c.$

(viii) $-\left(\dfrac{\cot^6 x}{6}+\dfrac{\cot^4 x}{4}\right)+c.$

(ix) $\dfrac{1}{(a^2-b^2)\sqrt{a^2\sin^2 x+b^2\cos^2 x}}+c.$

[**Hint :** put $a^2\sin^2 x+b^2\cos^2 x=t$]

(x) $-2 \operatorname{cosec} a\sqrt{\cos a - \tan x \, . \sin a} + c.$

[**Hint. :** Take $\sec^2 x$ as numerator]

(xi) $\tan x - \cot x - 3x + c.$

(xii) $\sin^{-1} (\sin x - \cos x) + c.$

42. (i) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + c.$ [**Hint :** put $x^2 = t$]

(ii) $\log\left|\frac{2\log x + 1}{3\log x + 2}\right| + C$ [**Hint :** put $\log x = t$]

(iii) $\frac{1}{\sqrt{5}} \log\left|\frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x}\right| + c$

(iv) $\sin^{-1}\left(\frac{x - 4}{5}\right) + c.$

(v) $2\log\left|\sqrt{x - a} + \sqrt{x - b}\right| + c$

(vi)

$$-\cos\alpha \sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right) - \sin\alpha . \log\left|\sin x + \sqrt{\sin^2 x - \sin^2 \alpha}\right| + c$$

$$\left[\textbf{Hint} : \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} = \frac{\sin(x-\alpha)}{\sin^2 x - \sin^2 \alpha}\right]$$

(vii) $\frac{5}{6}\log\left|3x^2 + 2x + 1\right| + \frac{(-11)}{3\sqrt{2}} \tan^{-1}\left(\frac{3x + 1}{\sqrt{2}}\right) + c$

(viii) $x - 3\log\left|x^2 + 6x + 12\right| + 2\sqrt{3} \tan^{-1}\left(\frac{x + 3}{\sqrt{3}}\right) + c$

(ix) $-\sqrt{4x - x^2} + 4\sin^{-1}\left(\frac{x - 2}{2}\right) + c$

(x) $\dfrac{-1}{3}\left(1 + x - x^2\right)^{\frac{3}{2}} + \dfrac{1}{8}(2x - 1)\sqrt{1 + x - x^2}$

$$+ \frac{5}{16}\sin^{-1}\left(\frac{2x - 1}{\sqrt{5}}\right) + c$$

(xi) $\left(x^2 + x + 1\right)^{\frac{3}{2}} - \dfrac{7}{2}\left[\begin{array}{l}\left(x + \dfrac{1}{2}\right)\sqrt{x^2 + x + 1} \\ + \dfrac{3}{8}\log\left|x + \dfrac{1}{2} + \sqrt{x^2 + x + 1}\right|\end{array}\right] + c$

(xii) $-\log\left|\cos x + \dfrac{1}{2} + \sqrt{\cos^2 x + \cos x}\right| + c$

[**Hint :** Multiply and divide by $\sqrt{\sec x + 1}$]

43. (i) $\dfrac{1}{7}\log\left|\dfrac{x^7}{x^7 + 1}\right| + c$

(ii) $\log\left|\dfrac{1 + \cos x}{2 + 3\cos x}\right| + c$

(iii) $\dfrac{-2}{3}\log|\cos\theta - 2| - \dfrac{1}{3}\log|1 + \cos\theta| + c.$

(iv) $\dfrac{9}{10}\log|x + 3| + \dfrac{4}{15}\log|x - 2| - \dfrac{1}{6}|x + 1| + c$

(v) $x + 4\log\left|\dfrac{(x - 2)^2}{x - 1}\right| + c$

(vi) $x + \dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right) - 3\tan^{-1}\left(\dfrac{x}{2}\right) + c$

[**Hint :** put $x^2 = t$]

(vii) $\dfrac{2}{17}\log|2x + 1| - \dfrac{1}{17}\log\left|x^2 + 4\right| + \dfrac{1}{34}\tan^{-1}\dfrac{x}{2} + c$

(viii) $-\frac{1}{2}\log|1-\cos x| - \frac{1}{6}\log|1+\cos x| + \frac{2}{3}\log|1-2\cos x| + c$

[**Hint :** Multiply N^r and D^r by $\sin x$ and put $\cos x = t$]

(ix) $\frac{-1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + \frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + c$

(x) $\frac{1}{2}\log\left|\frac{x^2 - x + 1}{x^2 + x + 1}\right| + c$

(xi) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1}\right| + c$

(xii) $\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 9}{3\sqrt{2}}\right) + c$

44. (i) $\frac{1}{3}\left[-x^3\cos x^3 + \sin x^3\right] + c$

(ii) $\frac{1}{2}\left[\sec x\tan x + \log|\sec x + \tan x|\right] + c$

[**Hint :** Write $\sec^3 x = \sec x \,.\, \sec^2 x$ and take $\sec x$ as first function]

(iii) $\frac{e^{ax}}{a^2 + b^2}\left[a\cos(bx + c) + b\sin(bx + c)\right] + c_1$

(iv) $2x\tan^{-1}3x - \frac{1}{3}\log\left|1 + 9x^2\right| + c$ [**Hint :** put $3x = \tan\theta$]

(v) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

(vi) $\left(\frac{x^4 - 1}{4}\right)\tan^{-1}x - \frac{x^3}{12} + \frac{x}{4} + c.$

(vii) $\frac{1}{2}e^{2x}\tan x + c.$ (viii) $\frac{e^x}{2x} + c.$

(ix) $\frac{x-a}{2}\sqrt{2ax - x^2} - \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + c$

(x) $e^x\left(\frac{x-1}{x+1}\right) + c.$

(xi) $e^x \tan x + c.$

(xii) $x \log|\log x| - \frac{x}{\log x} + c.$ [**Hint :** put $\log x = t \Rightarrow x = e^t$]

(xiii) $-2(6 + x - x^2)^{3/2}$
$+ 8\left[\frac{2x-1}{4}\sqrt{6 + x - x^2} + \frac{25}{8}\sin^{-1}\left(\frac{2x-1}{5}\right)\right] + c$

(xiv) $\frac{1}{2}(x+2)\sqrt{x^2 - 9} - \frac{3}{2}\log\left|x + \sqrt{x^2 - 9}\right| + c$

(xv) $\frac{2}{3}(x^2 - 4x + 3)^{3/2} - \left(\frac{x-2}{2}\right)\sqrt{x^2 - 4x + 3}$
$+ \frac{1}{2}\log\left|x - 2 + \sqrt{x^2 - 4x + 3}\right| + c$

(xvi) $\left(\frac{x-2}{2}\right)\sqrt{x^2 - 4x + 8} + 2\log\left|(x-2) + \sqrt{x^2 - 4x + 8}\right| + c$

45. (i) $\frac{1}{20}\log 3.$ (ii) $-\frac{\pi}{4}$

(iii) $\frac{\pi}{4} - \frac{1}{2}$. [**Hint :** put $x^2 = t$]

(iv) $\frac{\pi}{4} - \frac{1}{2}\log 2$.

(v) $\frac{\pi}{2}$.

(vi) $5 - 10\log\frac{15}{8} + \frac{25}{2}\log\left(\frac{6}{5}\right)$.

(vii) $\pi/2$.

$$\left[\textbf{Hint :} \left(\frac{x}{1+\cos x} + \frac{\sin x}{1+\cos x}\right) dx.\right]$$

46. (i) 8.

(ii) π.

(iii) $\frac{\pi}{8}\log 2$.

(iv) $\frac{-\pi}{2}\log 2$.

(v) $\frac{1}{4}\pi^2$.

(vi) 95/12.

$$\left[\textbf{Hint :} \int_{-2}^{2} f(x)\,dx = \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx\right]$$

(vii) $\frac{\pi^2}{16}$.

(viii) $\frac{\pi^2}{2ab}$.

$$\left[\textbf{Hint : Use} \int_{0}^{a} f(x) = \int_{0}^{a} f(a-x)\right]$$

47. (i) $\frac{\pi}{12}$.

(ii) $\frac{\pi}{2} - \log 2$.

(iii) 0.

(iv) $\pi/2$.

(v) $\dfrac{\pi^2}{4}$ (vi) $a\pi$.

48. $\dfrac{1}{2}$

49. $-x \cos x + \sin x + c$.

50. $x + \log x + c$.

51. $\dfrac{1}{2}\log|\sec x + \tan x| + c$.

52. $-\dfrac{1}{2}\left(\dfrac{\sin 3x}{3} - \sin x\right)$

53. $2 - \sqrt{2}$

54. 0

55. $\log|1 + \sin x| + c$

56. $x - \sin x + c$

57. $\log|\cos x + \sin x| + c$

58. $\dfrac{(a/c)^x}{\log(a/c)} + \dfrac{(b/c)^x}{\log(b/c)} + C$.

59. (i) $\dfrac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \dfrac{2\sqrt{x-x^2}}{\pi} - x + c$

(ii) $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + c$

(iii) $-\dfrac{1}{3}\left(1 + \dfrac{1}{x^2}\right)^{3/2}\left[\log\left(1 + \dfrac{1}{x^2}\right) - \dfrac{2}{3}\right] + c$

(iv) $\dfrac{\sin x - x\cos x}{x\sin x + \cos x} + c$

(v) $(x + a)\tan^{-1}\sqrt{\dfrac{x}{a}} - \sqrt{ax} + c$

(vi) $2\sin^{-1}\dfrac{\sqrt{3} - 1}{2}$

(vii) 0

(viii) $-\sqrt{2} - \sqrt{3} + 5$

(ix) $\dfrac{3}{\pi} + \dfrac{1}{\pi^2}$.

60. (i) $x - 4\log|x| + \dfrac{5}{4}\log|x - 1| + \dfrac{3}{4}\log|x + 1|$

$+ \log\left|x^2 + 1\right| - \dfrac{1}{2}\tan^{-1} x + c.$

$x + \dfrac{1}{2}\log\left|\dfrac{x - 1}{x + 1}\right| - \dfrac{1}{2}\tan^{-1} x + \log\left|\dfrac{x^2 - 1}{x^4 + 1}\right| + c.$

(ii) $\dfrac{1}{5}\log|x - 1| - \dfrac{1}{10}\log\left|x^2 + 4\right| - \dfrac{1}{10}\tan^{-1}\left(\dfrac{x}{2}\right) + c.$

(iii) $2x - \dfrac{1}{8}\log|x + 1| + \dfrac{81}{8}\log|x - 3| - \dfrac{27}{2(x - 3)} + c.$

(iv) $x + \dfrac{1}{2}\log\left|\dfrac{x - 2}{x + 2}\right| - \tan^{-1}\left(\dfrac{x}{2}\right) + c.$

(v) $\pi/\sqrt{2}$.

(vi) $\dfrac{1}{2\sqrt{2}}\tan^{-1}\dfrac{(x^2 - 1)}{\sqrt{2}x} - \dfrac{1}{4\sqrt{2}}\log\left|\dfrac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right| + c$

(vii) $\pi/8$.

61. (i) 14. (ii) $\frac{26}{3}$.

(iii) 26.

(iv) $\frac{1}{2}\left(127 + e^{8}\right)$.

(v) $\frac{141}{2}$.

62. (i) $\frac{\pi}{2} - \log 2$

(ii) $-\frac{1}{5}\log\left|\frac{\tan x - x}{2\tan x + 1}\right| + c$

(iii) $\frac{\pi}{8}\log 2$.

(iv) $\frac{\pi}{2}\log\left(\frac{1}{2}\right)$.

63. $\frac{1}{6}\log|1-\cos x| + \frac{1}{2}\log(1+\cos x) - \frac{2}{3}\log|1+2\cos x| + c$.

64. $3\log|(2-\sin\theta)| + \frac{4}{2-\sin\theta} + c$.

65. $\frac{1}{2}\sec x + \tan x + \frac{1}{2}\log|\sec x + \tan x| + c$.

66. $\frac{e^{2x}}{13}(2\cos 3x + 3\sin 3x) + c$.

CHAPTER 8

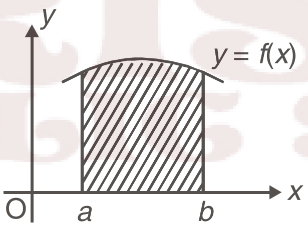
APPLICATIONS OF INTEGRALS

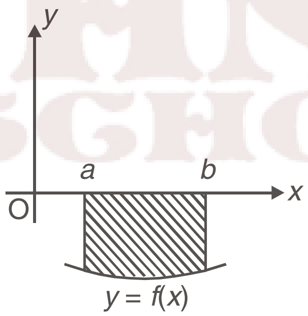
POINTS TO REMEMBER

AREA OF BOUNDED REGION

- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

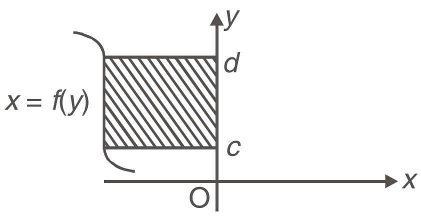
$$\text{Area} = \int_a^b |f(x)|\, dx$$

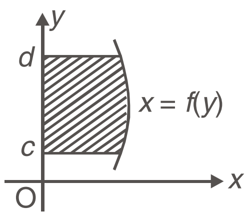




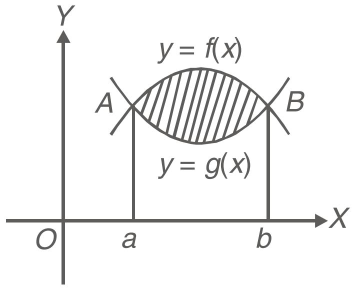
- Area bounded by the curve $x = f(y)$ the y-axis and between abscissas, $y = c$ and $y = d$ is given by

$$\text{Area} = \int_c^d |f(y)|\, dy$$





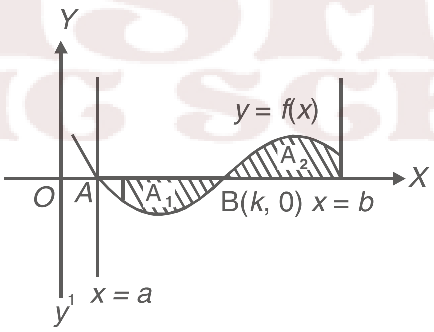
- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [a, b]$ and between the ordinate at $x = a$ and $x = b$ is given by



$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx$$

- Required Area

$$= \left| \int_a^k f(x)dx \right| + \int_k^b f(x)dx.$$



LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Find the area enclosed by circle $x^2 + y^2 = a^2$.
2. Find the area of region bounded by $\left\{(x, y) : |x - 1| \le y \le \sqrt{25 - x^2}\right\}$.
3. Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

4. Find the area of region in the first quadrant enclosed by x–axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

5. Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

6. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.

7. Find smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line

   $bx + ay = ab.$

8. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

9. Using integration, find the area of the region bounded by the triangle whose vertices are

   (a) (–1, 0), (1, 3) and (3, 2) (b) (–2, 2) (0, 5) and (3, 2)

10. Using integration, find the area bounded by the lines.

    (i) $x + 2y = 2$, $y - x = 1$ and $2x + y - 7 = 0$

    (ii) $y = 4x + 5$, $y = 5 - x$ and $4y - x = 5$.

11. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

12. Find the area of the region bounded by

    $y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \frac{3\pi}{2}$ and x-axis.

14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x-axis.

15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

16. Find area of smaller region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.

17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.

18. Using integration find the area of region in first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

19. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.

20. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

21. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $\left(0, \frac{\pi}{2}\right)$.

22. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5|\, dx$.

23. Find area enclosed between the curves, $y = 4x$ and $x^2 = 6y$.

24. Using integration, find the area of the following region :

$$\left\{(x, y) : |x - 1| \le y \le \sqrt{5 - x^2}\right\}$$

ANSWERS

1. πa^2 sq. units.

2. $\left(25\frac{\pi}{4} - \frac{1}{2}\right)$ *sq. units.*

3. πab sq. units

4. $(4\pi - 8)$ sq. units

5. $\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$ sq. units

7. $\frac{(\pi - 2)\,ab}{4}$ sq. units

8. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units

9. (a) 4 sq. units (b) 2 sq. units

10. (a) 6 sq. unit [**Hint.** Coordinate of vertices are (0, 1) (2, 3) (4, – 1)]

(b) $\frac{15}{2}$ sq. [**Hint :** Coordinate of vertices are (– 1, 1) (0, 5) (3, 2)]

11. $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

12. 1 sq. units

13. 3 sq. units

14. $\frac{25}{2}\pi$ sq. units

15. $\frac{1}{3}$ sq. units

16. $\frac{3}{2}(\pi - 2)$ sq. units

17. $\frac{9}{8}$ sq. units

18. $\frac{\pi}{3}$ sq. unit

19. 2π sq. unit.

20. $\frac{4}{3}(8 + 3\pi)$ sq. units

21. $\left(2 - \sqrt{2}\right)$ sq. units.

22. 13 sq. units.

23. 8 sq. units.

24. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units

CHAPTER 9

DIFFERENTIAL EQUATIONS

POINTS TO REMEMBER

- **Differential Equation :** Equation containing derivatives of a dependant variable with respect to an independent variable is called differential equation.

- **Order of a Differential Equation :** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.

- **Degree of a Differential Equation :** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.

- **Formation of a Differential Equation :** We differentiate the family of curves as many times as the number of arbitrary constant in the given family of curves. Now eliminate the arbitrary constants from these equations. After elimination the equation obtained is differential equation.

- **Solution of Differential Equation**

  (i) **Variable Separable Method**

  $$\frac{dy}{dx} = f(x, y)$$

  We separate the variables and get

  $$f(x)dx = g(y)dy$$

  Then $\int f(x)\,dx = \int g(y)\,dy + c$ is the required solutions.

  (ii) ***Homogenous Differential Equation :*** A differential equation of the form $\dfrac{dy}{dx} = \dfrac{f(x, y)}{g(x, y)}$ where $f(x, y)$ and $g(x, y)$ are both homogeneous functions of the same degree in x and y *i.e.,* of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is called a homogeneous differential equation.

For solving this type of equations we substitute $y = vx$ and then $\frac{dy}{dx} = v + x\frac{dv}{dx}$. The equation can be solved by variable separable method.

(iii) ***Linear Differential Equation :*** An equation of the from $\frac{dy}{dx} + Py = Q$ where P and Q are constant or functions of x only is called a linear differential equation. For finding solution of this type of equations, we find integrating factor $(I.F.) = e^{\int P\,dx}$.

Solution is $y\,(I.F.) = \int Q.\,(I.F.)\,dx + c$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the order and degree of the following differential equations.

(i) $\frac{dy}{dx} + \cos y = 0.$ (ii) $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4.$

(iii) $\frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5.$ (iv) $\frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0.$

(v) $\sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}.$ (vi) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}.$

(vii) $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x.$ (viii) $\frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$

2. Write the general solution of following differential equations.

(i) $\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}.$ (ii) $(e^x + e^{-x})\ dy = (e^x - e^{-x})dx$

(iii) $\frac{dy}{dx} = x^3 + e^x + x^e.$ (iv) $\frac{dy}{dx} = 5^{x+y}.$

(v) $\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}.$ (vi) $\frac{dy}{dx} = \frac{1 - 2y}{3x + 1}.$

3. Write integrating factor of the following differential equations

(i) $\frac{dy}{dx} + y \cos x = \sin x$

(ii) $\frac{dy}{dx} + y \sec^2 x = \sec x + \tan x$

(iii) $x^2 \frac{dy}{dx} + y = x^4.$ (iv) $x \frac{dy}{dx} + y \log x = x + y$

(v) $x \frac{dy}{dx} - 3y = x^3$ (vi) $\frac{dy}{dx} + y \tan x = \sec x$

(vii) $\frac{dy}{dx} + \frac{1}{1 + x^2} y = \sin x$

4. Write order of the differential equation of the family of following curves

(i) $y = Ae^x + Be^{x + c}$ (ii) $Ay = Bx^2$

(iii) $(x - a)^2 + (y - b)^2 = 9$ (iv) $Ax + By^2 = Bx^2 - Ay$

(v) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0.$ (vi) $y = a \cos (x + b)$

(vii) $y = a + be^{x+c}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

5. (i) Show that $y = e^{m \sin^{-1} x}$ is a solution of

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

(ii) Show that $y = \sin(\sin x)$ is a solution of differential equation

$$\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} + y\cos^2 x = 0.$$

(iii) Show that $y = Ax + \frac{B}{x}$ is a solution of $\frac{x^2d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$

(iv) Show that $y = a \cos(\log x) + b \sin(\log x)$ is a solution of

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

(v) Verify that $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the differential equation :

$$\left(a^2 + x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$$

(vi) Find the differential equation of the family of curves

$y = e^x(A \cos x + B \sin x)$, where A and B are arbitrary constants.

(vii) Find the differential equation of an ellipse with major and minor axes $2a$ and $2b$ respectively.

(viii) Form the differential equation representing the family of curves $(y - b)^2 = 4(x - a)$.

6. Solve the following differential equations.

(i) $\frac{dy}{dx} + y\cot x = \sin 2x.$ (ii) $x\frac{dy}{dx} + 2y = x^2 \log x.$

(iii) $\dfrac{dx}{dy} + \dfrac{1}{x} . y = \cos x + \dfrac{\sin x}{x}, \quad x > 0.$

(iv) $\cos^3 x \dfrac{dy}{dx} + \cos x = \sin x.$

(v) $ydx + \left(x - y^3\right) dy = 0$

(vi) $ye^y dx = \left(y^3 + 2xe^y\right) dy$

7. Solve each of the following differential equations :

(i) $y - x\dfrac{dy}{dx} = 2\left(y^2 + \dfrac{dy}{dx}\right).$

(ii) $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0.$

(iii) $x\sqrt{1 - y^2} dy + y\sqrt{1 - x^2} dx = 0.$

(iv) $\sqrt{\left(1 - x^2\right)\left(1 - y^2\right)} \, dy + xy \, dx = 0.$

(v) $(xy^2 + x) \, dx + (yx^2 + y) \, dy = 0; \; y(0) = 1.$

(vi) $\dfrac{dy}{dx} = y \sin^3 x \cos^3 x + xy \, e^x.$

(vii) $\tan x \; \tan y \, dx + \sec^2 x \sec^2 y \, dy = 0$

8. Solve the following differential equations :

(i) $x^2 \, y \, dx - (x^3 + y^3) \, dy = 0.$

(ii) $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2.$

(iii) $\left(x^2 - y^2\right) dx + 2xy \; dy = 0, \quad y(1) = 1.$

(iv) $\left(y \sin \frac{x}{y}\right) dx = \left(x \sin \frac{x}{y} - y\right) dy.$ (v) $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right).$

(vi) $\frac{dy}{dx} = \frac{2xy}{x^2 + y^2}$ (vii) $\frac{dy}{dx} = e^{x+y} + x^2 e^y.$

(viii) $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}.$

(ix) $\left(3xy + y^2\right) dx + \left(x^2 + xy\right) dy = 0$

9. (i) Form the differential equation of the family of circles touching y-axis at (0, 0).

(ii) Form the differential equation of family of parabolas having vertex at (0, 0) and axis along the (i) positive y-axis (ii) positive x-axis.

(iii) Form differential equation of family of circles passing through origin and whose centre lie on x-axis.

(iv) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.

10. Show that the differential equation $\frac{dy}{dx} = \frac{x + 2y}{x - 2y}$ is homogeneous and solve it.

11. Show that the differential equation :

$(x^2 + 2xy - y^2)\, dx + (y^2 + 2xy - x^2)\, dy = 0$ is homogeneous and solve it.

12. Solve the following differential equations :

(i) $\frac{dy}{dx} - 2y = \cos 3x.$

(ii) $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$ if $y\left(\frac{\pi}{2}\right) = 1$

(iii) $3e^x \tan y\, dx + \left(1 - e^x\right) \sec^2 y\, dy = 0$

13. Solve the following differential equations :

(i) $(x^3 + y^3)\ dx = (x^2y + xy^2)dy.$

(ii) $x\ dy - y\ dx = \sqrt{x^2 + y^2}\,dx.$

(iii) $y\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\}dx$

$$- x\left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\}dy = 0.$$

(iv) $x^2dy + y(x + y)\ dx = 0$ given that $y = 1$ when $x = 1$.

(v) $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$ if $y(e) = 0$

(vi) $(x^3 - 3xy^2)\ dx = (y^3 - 3x^2y)dy.$

(vii) $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$ given that $y = 0$ when $x = 1$

16. Solve the following differential equations :

(i) $\cos^2 x\frac{dy}{dx} = \tan x - y.$

(ii) $x \cos x\frac{dy}{dx} + y\,(x \sin x + \cos x) = 1.$

(iii) $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0.$

(iv) $(y - \sin x)\ dx + \tan x\ dy = 0,\ y(0) = 0.$

LONG ANSWER TYPE QUESTIONS (6 MARKS EACH)

17. Solve the following differential equations :

(i) $(x\,dy - y\,dx)\,y \sin\left(\frac{y}{x}\right) = (y\,dx + x\,dy)\,x \cos\left(\frac{y}{x}\right)$

(ii) $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.

(iii) $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ given that $y(0) = 0$.

ANSWERS

1.(i) order = 1, degree = 1 (ii) order = 2, degree = 1

(iii) order = 4, degree = 1 (iv) order = 5, degree is not defined.

(v) order = 2, degree = 2 (vi) order = 2, degree = 2

(vii) order = 3, degree = 2 (viii) order = 1, degree is not defined

2.(i) $y = \frac{x^6}{6} + \frac{x^3}{6} - 2\log|x| + c$ (ii) $y = \log_e\left|e^x + e^{-x}\right| + c$

(iii) $y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c.$ (iv) $5^x + 5^{-y} = c$

(v) $2(y - x) + \sin 2y + \sin 2x = c.$ (vi) $2\log|3x + 1| + 3\log|1 - 2y| = c.$

3.(i) $e^{\sin x}$ (ii) $e^{\tan x}$

(iii) $e^{-1/x}$ (iv) $e^{\frac{(\log x)^2}{2}}$

(v) $\frac{1}{x^3}$ (vi) $\sec x$

(vii) $e^{\tan^{-1} x}$

4.(i) 1 (ii) 1

(iii) 2 (iv) 1

(v) 1 (vi) 1

(vii) 2

5.(vi) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$ (vii) $x\left(\dfrac{dy}{dx}\right)^2 + xy\dfrac{d^2y}{dx^2} = y\dfrac{dy}{dx}$

(viii) $2\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

6.(i) $y \sin x = \dfrac{2\sin^3 x}{3} + c$ (ii) $y = \dfrac{x^2\left(4\log_e x - 1\right)}{16} + \dfrac{c}{x^2}$

(iii) $y = \sin x + \dfrac{c}{x}, \ x > 0$ (iv) $y = \tan x - 1 + ce^{-\tan x}$

(v) $xy = \dfrac{y^4}{4} + c$ (vi) $x = -y^2e^{-y} + cy^2$

7.(i) $cy = (x + 2)(1 - 2y)$ (ii) $\left(e^x + 2\right)\sec y = c$

(iii) $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

(iv) $\dfrac{1}{2}\log\left|\dfrac{\sqrt{1 - y^2} - 1}{\sqrt{1 - y^2} + 1}\right| = \sqrt{1 - x^2} - \sqrt{1 - y^2} + c$

(v) $\left(x^2 + 1\right)\left(y^2 + 1\right) = 2$

(vi) $\log y = -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + xe^x - e^x + c$

$$= \frac{1}{16}\left[\frac{\cos^3 2x}{3} - \cos 2x\right] + (x-1)e^x + c$$

(vii) $\log|\tan y| - \frac{\cos 2x}{y} = c$

8.(i) $\frac{-x^3}{3y^3} + \log|y| = c$ (ii) $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

(iii) $x^2 + y^2 = 2x$

(iv) $y = ce^{\cos(x/y)}$ [**Hint :** Put $\frac{1}{x} = v$]

(v) $\sin\left(\frac{y}{x}\right) = cx$ (vi) $c\left(x^2 - y^2\right) = y$

(vii) $-e^{-y} = e^x + \frac{x^3}{3} + c$ (viii) $\sin^{-1} y = \sin^{-1} x + c$

(ix) $x\log\left(x^3 y\right) + y = cx$

9.(i) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$ (ii) $2y = x\frac{dy}{dx}$, $y = 2x\frac{dy}{dx}$

(iii) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(iv) $(x-y)^2(1+y')^2 = (x+yy')^2$

10. $\log\left|x^2 + xy + y^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3x}}\right) + c$

11. $\frac{x^3}{x^2+y^2} = \frac{c}{x}(x+y)$

12.(i) $y = \dfrac{3 \sin 3x}{13} - \dfrac{2 \cos 3x}{13} + ce^{2x}$ (ii) $y = \dfrac{2}{3}\sin^2 x + \dfrac{1}{3}\operatorname{cosec} x$

(iii) $\tan y = k\left(1 - e^x\right)^3$

13.(i) $-y = x \log\left\{c\left(x - y\right)\right\}$ (ii) $cx^2 = y + \sqrt{x^2 + y^2}$

(iii) $xy \cos\left(\dfrac{y}{x}\right) = c$ (iv) $3x^2y = y + 2x$

(v) $y = -x \log\left(\log|x|\right),\ x \neq 0$ (vi) $c\left(x^2 + y^2\right) = \sqrt{x^2 - y^2}$.

(vii) $\cos \dfrac{y}{x} = \log|x| + 1$

16. (i) $y = \tan x - 1 + ce^{\tan^{-1} x}$ (ii) $y = \dfrac{\sin x}{x} + c\,\dfrac{\cos x}{x}$

(iii) $x + ye^{\frac{x}{y}} = c$ (iv) $2y = \sin x$

17. (i) $c\,xy = \sec\left(\dfrac{y}{x}\right)$

(ii) $(1 - e)^3 \tan y = (1 - e^x)^3$

(iii) $y = x^2$.

CHAPTER 10

VECTORS

POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a *vector*. It is denoted by a directed line segment.
- Two or more vectors which are parallel to same line are called *collinear vectors*.
- *Position vector* of a point $P(a, b, c)$ w.r.t. origin (0, 0, 0) is denoted by $\overrightarrow{OP}$, where $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$ and $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$.
- If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space, then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ and

  $$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

- If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the two sides of a triangle taken in order, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by third side of triangle taken in opposite order. This is called triangle *law of addition of vectors.*
- If $\vec{a}$ is any vector and λ is a scalar, then $\lambda\vec{a}$ is a vector collinear with $\vec{a}$ and $|\lambda\vec{a}| = |\lambda||\vec{a}|$.
- If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{a} = \lambda\vec{b}$ where λ is some scalar.
- Any vector $\vec{a}$ can be written as $\vec{a} = |\vec{a}|\hat{a}$, where $\hat{a}$ is a unit vector in the direction of $\vec{a}$.

- If $\vec{a}$ and $\vec{b}$ be the position vectors of points *A* and *B,* and *C* is any point which divides $\overrightarrow{AB}$ in ratio *m* : *n* internally then position vector $\vec{c}$ of point *C* is given as $\vec{C} = \dfrac{m\vec{b} + n\vec{a}}{m + n}$. If *C* divides $\overrightarrow{AB}$ in ratio *m* : *n* externally, then $\vec{C} = \dfrac{m\vec{b} - n\vec{a}}{m - n}$.

- The angles α, β and γ made by $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with positive direction of x, *y* and *z*-axis are called direction angles and cosines of these angles are called *direction cosines* of $\vec{r}$ usually denoted as $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

  Also $l = \dfrac{a}{|\vec{r}|}$, $m = \dfrac{b}{|\vec{r}|}$, $n = \dfrac{c}{|\vec{r}|}$ and $l^2 + m^2 + n^2 = 1$.

- The numbers *a, b, c* proportional to *l, m, n* are called *direction ratios*.

- *Scalar product* of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a}.\vec{b}$ and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ $(0 \le \theta \le \pi)$.

- *Dot product* of two vectors is commutative *i.e.* $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

- $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} = \vec{o}, \vec{b} = \vec{o}$ or $\vec{a} \perp \vec{b}$.

- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$, so $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$.

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

  $\vec{a} \cdot \vec{b} = a_1a_2 + b_1b_2 + c_1c_2$.

- *Projection of* $\overrightarrow{a}$ on $\overrightarrow{b} = \left|\dfrac{\overrightarrow{a} \,.\, \overrightarrow{b}}{|\overrightarrow{b}|}\right|$ and projection vector of

  $$\overrightarrow{a} \text{ along } \overrightarrow{b} = \left(\frac{(\overrightarrow{a} \,.\, \overrightarrow{b})}{|\overrightarrow{b}|}\right)\hat{b}.$$

- *Cross product* or vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\overrightarrow{a}||\overrightarrow{b}|\sin\theta\,\hat{n}$. were θ is the angle between $\vec{a}$ and $\vec{b}$ $(0 \le \theta \le \pi)$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}, \vec{b}$ *and* $\hat{n}$ form a right handed system.

- Cross product of two vectors is not commutative i.e., $\vec{a} \times \vec{b} \ne \vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b} = -\left(\vec{b} \times \vec{a}\right)$.

- $\vec{a} \times \vec{b} = \vec{o} \Leftrightarrow \vec{a} = \vec{o},\ \vec{b} = \vec{o}$ *or* $\vec{a} \parallel \vec{b}$.

- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{o}$.

- $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times i = -\hat{k},\ \hat{k} \times \hat{j} = -\hat{i},\ \hat{i} \times \hat{k} = -\hat{j}$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm\left(\dfrac{\left(\vec{a} \times \vec{b}\right)}{\left|\vec{a} \times \vec{b}\right|}\right)$.

- $\left|\vec{a} \times \vec{b}\right|$ is the *area of parallelogram* whose adjacent sides are $\vec{a}$ and $\vec{b}$.

- $\dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$ is the area of parallelogram where diagonals are $\vec{a}$ and $\vec{b}$.

- If $\vec{a}$, $\vec{b}$ and $\vec{c}$ forms a triangle, then *area of the triangle.*

$$= \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = \frac{1}{2}\left|\vec{b} \times \vec{c}\right| = \frac{1}{2}\left|\vec{c} \times \vec{a}\right|.$$

- Scalar triple product of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is defined as $\vec{a}.\left(\vec{b} \times \vec{c}\right)$ and is denoted as $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

- Geometrically, absolute value of scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ represents volume of a parallelepiped whose coterminous edges are $\vec{a}$, $\vec{b}$ and $\vec{c}$.

- $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar $\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

- $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ &

  $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- The scalar triple product of three vectors is zero if any two of them are same or collinear.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What are the horizontal and vertical components of a vector $\vec{a}$ of magnitude 5 making an angle of 150° with the direction of x-axis.

2. What is $a \in R$ such that $|a\vec{x}| = 1$, where $\vec{x} = \hat{i} - 2\hat{j} + 2\hat{k}$?

3. When is $|\vec{x} + \vec{y}| = |\vec{x}| + |\vec{y}|$?

4. What is the area of a parallelogram whose sides are given by $2\hat{i} - \hat{j}$ and $\hat{i} + 5\hat{k}$?

5. What is the angle between $\vec{a}$ *and* $\vec{b}$, If

   $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$.

6. Write a unit vector which makes an angle of $\frac{\pi}{4}$ with x-axis and $\frac{\pi}{3}$ with z-axis and an acute angle with y-axis.

7. If A is the point (4, 5) and vector $\overrightarrow{AB}$ has components 2 and 6 along x-axis and y-axis respectively then write point B.

8. What is the point of trisection of PQ nearer to P if positions of P and Q are $3\hat{i} + 3\hat{j} - 4\hat{k}$ and $9\hat{i} + 8\hat{j} - 10\hat{k}$ respectively?

9. Write the vector in the direction of $2\hat{i} + 3\hat{j} + 2\sqrt{3}\hat{k}$, whose magnitude is 10 units.

10. What are the direction cosines of a vector equiangular with co-ordinate axes?

11. What is the angle which the vector $3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the x-axis?

12. Write a unit vector perpendicular to both the vectors

$$3\hat{i} - 2\hat{j} + \hat{k} \text{ and } -2\hat{i} + \hat{j} - 2\hat{k}.$$

13. What is the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$?

14. If $|\vec{a}| = 2$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{a} \perp \vec{b}$, what is the value of $|\vec{a} + \vec{b}|$?

15. For what value of λ, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ is perpendicular to $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$?

16. What is $|\vec{a}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 3$ and $2|\vec{b}| = |\vec{a}|$?

17. What is the angle between $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$?

18. In a parallelogram $ABCD$, $\overrightarrow{AB} = 2\hat{i} - \hat{j} + 4\hat{k}$ *and* $\overrightarrow{AC} = \hat{i} + \hat{j} + 4\hat{k}$. What is the length of side BC?

19. What is the area of a parallelogram whose diagonals are given by vectors $2\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{k}$?

20. Find $|\vec{x}|$ if for a unit vector $\hat{a}$, $(\vec{x} - \hat{a}).(\vec{x} + \hat{a}) = 12$.

21. If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\vec{a} + \vec{b}$ is also a unit vector then what is the angle between $\vec{a}$ and $\vec{b}$?

22. If $\hat{i}, \hat{j}, \hat{k}$ are the usual three mutually perpendicular unit vectors then what is the value of $\hat{i}\,.\,(\hat{j} \times \hat{k}) + \hat{j}\,.(\hat{i} \times \hat{k}) + \vec{k}.(\hat{j} \times \hat{i})$?

23. What is the angle between $\vec{x}$ and $\vec{y}$ if $\vec{x}\,.\,\vec{y} = |\vec{x} \times \vec{y}|$?

24. Write a unit vector in *xy*-plane, making an angle of 30° with the +ve direction of *x*–axis.

25. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors with $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then what is the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$?

26. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a} + 2\vec{b}\right)$ is perpendicular to $\left(5\vec{a} - 4\vec{b}\right)$, then what is the angle between $\vec{a}$ and $\vec{b}$?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

27. If ABCDEF is a regular hexagon then using triangle law of addition prove that :

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$$

O being the centre of hexagon.

28. Points *L, M, N* divides the sides *BC, CA, AB* of a ΔABC in the ratios 1 : 4, 3 : 2, 3 : 7 respectively. Prove that $\overrightarrow{AL} + \overrightarrow{BM} + \overrightarrow{CN}$ is a vector parallel to $\overrightarrow{CK}$ where *K* divides *AB* in ratio 1 : 3.

29. The scalar product of vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ *and* $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ.

30. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Show that $\vec{a} + \vec{b} + \vec{c}$ makes equal angles with $\vec{a}$, $\vec{b}$ and $\vec{c}$ with each angle as $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

31. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

32. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

33. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

34. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find a vector $\vec{d}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 1$.

35. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ are the given vectors then find a vector $\vec{b}$ satisfying the equation $\vec{a} \times \vec{b} = \vec{c}$, $\vec{a} \cdot \vec{b} = 3$.

36. Find a unit vector perpendicular to plane *ABC,* when position vectors of *A, B, C* are $3\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} - \hat{j} - 3\hat{k}$ and $4\hat{i} - 3\hat{j} + \hat{k}$ respectively.

37. For any two vector, show that $\left|\vec{a} + \vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$.

38. Evaluate $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{j}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2$.

39. If $\hat{a}$ and $\hat{b}$ are unit vector inclined at an angle θ than prove that :

(i) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|$. (ii) $\tan\frac{\theta}{2} = \left|\frac{\hat{a} - \hat{b}}{\hat{a} + \hat{b}}\right|$.

40. For any two vectors, show that $\left|\vec{a} \times \vec{b}\right| = \sqrt{a^2 b^2 - \left(\vec{a} \cdot \vec{b}\right)^2}$.

41. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{j}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then find the value of *x*.

42. Prove that angle between any two diagonals of *a* cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

43. Let $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$.

44. Prove that the normal vector to the plane containing three points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ lies in the direction of vector $\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}$.

45. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices *A, B, C* of a triangle *ABC* then show that the area of ΔABC is $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$.

46. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then prove that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ provided $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

47. Dot product of a vector with vectors $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ is 0, 5 and 8 respectively. Find the vectors.

48. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - \lambda\hat{k}$, find λ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

49. Let $\vec{a}$ and $\vec{b}$ be vectors such that $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} - \vec{b}\right| = 1$, then find $\left|\vec{a} + \vec{b}\right|$.

50. If $\left|\vec{a}\right| = 2$, $\left|\vec{b}\right| = 5$ and $\vec{a} \times \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the value of $\vec{a} \cdot \vec{b}$.

51. $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{b} \times \vec{c} = \vec{a}$ and $\vec{a} \times \vec{b} = \vec{c}$. Prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular to each other and $\left|\vec{b}\right| = 1$, $\left|\vec{c}\right| = \left|\vec{a}\right|$.

52. If $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ *and* $\vec{c} = 3\hat{i} - \hat{k}$ find $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

53. Find volume of parallelepiped whose coterminous edges are given by vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$.

54. Find the value of λ such that $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar.

55. Show that the four points (–1, 4, –3), (3, 2, –5) (–3, 8, –5) and (–3, 2, 1) are coplanar.

56. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that

$$\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a} \ \vec{b} \ \vec{c}\right]$$

57. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, prove that $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.

ANSWERS

1. $-\frac{5\sqrt{3}}{2}, \frac{5}{2}$.

2. $a = \pm\frac{1}{3}$

3. $\vec{x}$ and $\vec{y}$ are like parallel vectors.

4. $\sqrt{126}$ sq units.

5. $\frac{\pi}{3}$

6. $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

7. (6, 11)

8. $\left(5, \frac{14}{3}, -6\right)$

9. $4\hat{i} + 6\hat{j} + 4\sqrt{3}\hat{k}$.

10. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

11. $\cos^{-1}\left(\frac{3}{7}\right)$.

12. $\frac{3\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{26}}$.

13. 0

14. 4

15. –9

16. 2

17. $\frac{\pi}{2}$.

18. $\sqrt{5}$

19. $\frac{3}{2}$ sq. units.

20. $\sqrt{13}$

21. $\frac{2\pi}{3}$

22. -1

23. $\frac{\pi}{4}$

24. $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

25. $-\frac{3}{2}$

26. $\frac{\pi}{3}$

29. $\lambda = 1$

31. $\overrightarrow{\beta} = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right).$

33. $60°$

34. $\frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}.$

35. $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}.$

36. $\frac{-1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right).$

38. $2\left|\overrightarrow{a}\right|^2$

41. $x = -2$

47. $\hat{i} + 2\hat{j} + \hat{k}$

48. $\pm\sqrt{73}$

49. $\sqrt{3}$

50. $\frac{91}{10}$

52. 4

53. 37

54. $\lambda = 1$

CHAPTER 11

THREE DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

- Distance between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

- (i) The coordinates of point R which divides line segment PQ where $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m : n$ internally are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)$.

  (ii) The co-ordinates of a point which divides join of (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio of $m : n$ externally are $\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n}\right)$.

- Direction ratios of a line through (x_1, y_1, z_1) and (x_2, y_2, z_2) are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

- Direction cosines of a line whose direction ratios are a, b, c are given by

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \ m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}, \ n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

- (i) Vector equation of a line through point $\vec{a}$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

  (ii) Cartesian equation of a line through point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}.$$

- (i) Vector equation of line through two points

  $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$.

  (ii) Cartesian equation of a line through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$.

- Angle 'θ' between lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by $\cos\theta = \dfrac{\vec{b_1} \cdot \vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}$.

- Angle θ between lines $\dfrac{x - x_1}{a_1} = \dfrac{y + y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_2} = \dfrac{y + y_2}{b_2} = \dfrac{z - z_2}{c_2}$ is given by

  $$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

- Two lines are perpendicular to each other if

  $\vec{b_1} \cdot \vec{b_2} = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

- Equation of plane :

  (i) At a distance of p unit from origin and perpendicular to $\hat{n}$ is $\vec{r} \cdot \hat{n} = p$ and corresponding Cartesian form is $lx + my + nz = p$ when l, m and n are *d.c.*s of normal to plane.

  (ii) Passing through $\vec{a}$ and normal to $\vec{n}$ is $\left(\vec{r} - \vec{a}\right).\vec{n} = 0$ and corresponding Cartesian form is $a(x - x_1) + b\,(y - y_1) + c(z - z_1) = 0$ where a, b, c are *d.r.*'s of normal to plane and (x_1, y_1, z_1) lies on the plane.

  (iii) Passing through three non collinear points is

  $\left(\vec{r} - \vec{a}\right) \cdot \left[\left(\vec{b} - \vec{a}\right) \times \left(\vec{c} - \vec{a}\right)\right] = 0$

$$\text{or} \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(iv) Having intercepts a, b and c on co-ordinate axis is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$.

(v) Planes passing through the line of intersection of planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ is

$$\left(\vec{r} \cdot \vec{n_1} - d_1\right) + \lambda\left(\vec{r} \cdot \vec{n_2} - d_2\right) = 0.$$

- (i) Angle 'θ' between planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ is given by $\cos\theta = \dfrac{\vec{n_1} \cdot \vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|}$.

(ii) Angle θ between $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

(iii) Two planes are perpendicular to each other iff $\vec{n_1} \cdot \vec{n_2} = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(iv) Two planes are parallel iff $\vec{n_1} = \lambda\vec{n_2}$ for some scaler $\lambda \neq 0$ or $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

- (i) Distance of a point $\left(\vec{a}\right)$ from plane $\left(\vec{r} \cdot \vec{n} = d\right)$ is $\dfrac{\left|\vec{a} \cdot \vec{n} - d\right|}{\left|\vec{n}\right|}$.

(ii) Distance of a point (x_1, y_1, z_1) from plane $ax + by + cz = d$ is $\dfrac{\left|ax_1 + by_1 + cz_1 - d\right|}{\left|a^2 + b^2 + c^2\right|}$.

12. (i) Two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are coplanar. Iff $\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = 0$ and equation of plane, containing these lines is $\left(\vec{r} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = 0$.

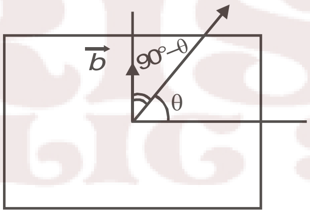
(ii) Two lines $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ are coplanar Iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

and equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

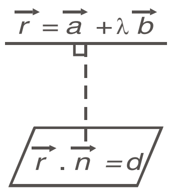
- (i) The angle θ between line $\vec{r} = \vec{a} + \lambda \vec{b}$ and plane $\vec{r} \cdot \vec{n} = d$ is given as $\sin\theta = \dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|}$.



(ii) The angle θ between line $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2 z = d$ is given as

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

(iii) A line $\vec{r} = \vec{a} + \lambda \vec{b}$ is parallel to plane $\vec{r} \cdot \vec{n} = d$ $\Leftrightarrow \vec{b} \cdot \vec{n} = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What is the distance of point (a, b, c) from x-axis?

2. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?

3. Write the equation of a line passing through (2, –3, 5) and parallel to line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$.

4. Write the equation of a line through (1, 2, 3) and perpendicular to $\vec{r} \cdot \left(\hat{i} - \hat{j} + 3\hat{k}\right) = 5$.

5. What is the value of λ for which the lines $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular to each other.

6. If a line makes angle α, β, and γ with co-ordinate axes, then what is the value of

   $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$?

7. Write line $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{j} - \hat{k}\right)$ into Cartesian form.

8. If the direction ratios of a line are 1, –2, 2 then what are the direction cosines of the line?

9. Find the angle between the planes $2x - 3y + 6z = 9$ and xy – plane.

10. Write equation of a line passing through (0, 1, 2) and equally inclined to co-ordinate axes.

11. What is the perpendicular distance of plane $2x - y + 3z = 10$ from origin?

12. What is the y-intercept of the plane $x - 5y + 7z = 10$?

13. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.

14. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes.

15. Are the planes $x + y - 2z + 4 = 0$ and $3x + 3y - 6z + 5 = 0$ intersecting?

16. What is the equation of the plane through the point (1, 4, – 2) and parallel to the plane $-2x + y - 3z = 7$?

17. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

18. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?

19. Find the angles between the planes $\vec{r} \cdot \left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$ and $\vec{r} \cdot \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0$.

20. What is the angle between the line $\dfrac{x+1}{3} = \dfrac{2y-1}{4} = \dfrac{2-z}{-4}$ and the plane $2x + y - 2z + 4 = 0$?

21. If O is origin $OP = 3$ with direction ratios proportional to –1, 2, – 2 then what are the coordinates of P?

22. What is the distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} + \hat{j} + 4\hat{k}\right)$ from the plane $\vec{r} \cdot \left(-\hat{i} + 5\hat{j} - \hat{k}\right) + 5 = 0$.

23. Write the line $2x = 3y = 4z$ in vector form.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

24. The line $\dfrac{x-4}{1} = \dfrac{2y-4}{2} = \dfrac{k-z}{-2}$ lies exactly in the plane $2x - 4y + z = 7$. Find the value of k.

25. Find the equation of a plane containing the points (0, –1, –1), (–4, 4, 4) and (4, 5, 1). Also show that (3, 9, 4) lies on that plane.

26. Find the equation of the plane which is perpendicular to the plane $\vec{r} \cdot \left(5\hat{i} + 3\hat{j} + 6\hat{k}\right) + 8 = 0$ & which is containing the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 4$ and $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$.

27. If l_1, m_1, n_1, and l_2, m_2, n_2 are direction cosines of two mutually perpendicular lines, show that the direction cosines of line perpendicular to both of them are

$$m_1 n_2 - n_1 m_2,\ n_1 l_2 - l_1 n_2,\ l_1 m_2 - m_1 l_2.$$

28. Find vector and Cartesian equation of a line passing through a point with position vectors $2\hat{i} + \hat{j} + \hat{k}$ and which is parallel to the line joining the points with position vectors $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.

29. Find the equation of the plane passing through the point (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$.

30. Find equation of plane through line of intersection of planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which is at a unit distance from origin.

31. Find the image of the point (3, –2, 1) in the plane $3x - y + 4z = 2$.

32. Find the equation of a line passing through (2, 0, 5) and which is parallel to line $6x - 2 = 3y + 1 = 2z - 2$.

33. Find image (reflection) of the point (7, 4, – 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

34. Find equations of a plane passing through the points (2, –1, 0) and (3, –4, 5) and parallel to the line $2x = 3y = 4z$.

35. Find distance of the point (– 1, – 5, – 10) from the point of intersection of line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.

36. Find equation of the plane passing through the points (2, 3, – 4) and (1, –1, 3) and parallel to the x–axis.

37. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

38. Find the equation of the plane passing through the intersection of two plane $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

39. Find the distance between the planes $2x + 3y - 4z + 5 = 0$ and $\vec{r}.(4\hat{i} + 6\hat{j} - 8\hat{k}) = 11$.

40. Find the equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ whose perpendicular distance from the point (1, 2, 3) is 1 unit.

41. Show that the lines $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7}$ and $\dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5}$ intersect each other. Find the point of intersection.

42. Find the shortest distance between the lines

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right).$$

43. Find the distance of the point (–2, 3, –4) from the line $\dfrac{x+2}{3} = \dfrac{2y+3}{4} = \dfrac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

44. Find the equation of plane passing through the point (–1, –1, 2) and perpendicular to each of the plane

$$\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 3\hat{k}\right) = 2 \text{ and } \vec{r} \cdot \left(5\hat{i} - 4\hat{j} + \hat{k}\right) = 6.$$

45. Find the equation of a plane passing through (–1, 3, 2) and parallel to each of the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ and $\dfrac{x+2}{-3} = \dfrac{y-1}{2} = \dfrac{z+1}{5}$.

46. Show that the plane $\vec{r} \cdot \left(\hat{i} - 3\hat{j} + 5\hat{k}\right) = 7$ contains the line $\vec{r} = \left(\hat{i} + 3\hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} + \hat{j}\right)$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

47. Check the coplanarity of lines

$$\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-3\hat{i} + \hat{j} + 5\hat{k}\right)$$

$$\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \mu\left(-\hat{i} + 2\hat{j} + 5\hat{k}\right)$$

If they are coplanar, find equation of the plane containing the lines.

48. Find shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

49. Find shortest distance between the lines :

$$\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\hat{\lambda})\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (2\mu + 1)\hat{k}.$$

50. A variable plane is at a constant distance $3p$ from the origin and meets the coordinate axes in A, B and C. If the centroid of ΔABC is (α, β, γ), then show that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$.

51. A vector $\vec{n}$ of magnitude 8 units is inclined to x–axis at 45°, y axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$, find its equation in vector form.

52. Find the foot of perpendicular from the point $2\hat{i} - \hat{j} + 5\hat{k}$ on the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

53. A line makes angles α, β, λ, δ with the four diagonals of a cube. Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

54. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy-plane.

ANSWERS

1. $\sqrt{b^2 + c^2}$

2. 90°

3. $\dfrac{x-2}{3} = \dfrac{y+3}{4} = \dfrac{z-5}{-1}.$

4. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 3\hat{k})$

5. $\lambda = 2$

6. 2

7. $\dfrac{x-1}{0} = \dfrac{y+1}{2} = \dfrac{z}{-1}.$

8. $\pm\dfrac{1}{\sqrt{3}}, \mp\dfrac{2}{\sqrt{3}}, \pm\dfrac{2}{\sqrt{3}}$

9. $\cos^{-1}$ (6/7).

10. $\dfrac{x}{a} = \dfrac{y-1}{a} = \dfrac{z-2}{a}$, $a \in R - \{0\}$

11. $\dfrac{10}{\sqrt{14}}$

12. –2

13. $\dfrac{1}{6}$

14. $x + y + z = 1$

15. No

16. $-2x + y - 3z = 8$

17. $\vec{r} \cdot \left(2\hat{i} + \hat{j} + 2\hat{k}\right) = 24$

18. $2x + 3y + 4z = 29$

19. $\cos^{-1}\left(\dfrac{11}{21}\right)$

20. 0 (line is parallel to plane)

21. (–1, 2, –2)

22. $\dfrac{10}{3\sqrt{3}}$

23. $\vec{r} = \vec{o} + \lambda\left(6\hat{i} + 4\hat{j} + 3\hat{k}\right)$

24. $k = 7$

25. $5x - 7y + 11z + 4 = 0$.

26. $\vec{r} \cdot \left(-51\hat{i} - 15\hat{j} + 50\hat{k}\right) = 173$

28. $\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\dfrac{x-2}{2} = \dfrac{y+1}{-2} = \dfrac{z-1}{1}$.

29. $x - 2y + 3z = 1$

30. $\vec{r} \cdot \left(8\hat{i} + 4\hat{j} + 8\hat{k}\right) + 12 = 0$ or $\vec{r} \cdot \left(-4\hat{i} + 8\hat{j} - 8\hat{k}\right) + 12 = 0$

31. (0, –1, –3)

32. $\dfrac{x-2}{1} = \dfrac{y}{2} = \dfrac{z-5}{3}$.

33. $\left(\dfrac{47}{7}, -\dfrac{18}{7}, \dfrac{43}{7}\right)$

34. $29x - 27y - 22z = 85$

35. 13

36. $7y + 4z = 5$

37. 1

38. $x - 20y + 27z = 14$

39. $\frac{21}{2\sqrt{29}}$ units.

40. $x - 2y + 2z = 0$ or $x - 2y + 2z = 6$

41. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

42. $\frac{1}{\sqrt{6}}$

43. $\frac{17}{2}$

44. $\vec{r} \cdot \left(9\hat{i} + 17\hat{j} + 23\hat{k}\right) = 20$

45. $2x - 7y + 4z + 15 = 0$

47. $x - 2y + z = 0$

48. $\frac{16}{7}$

49. $\frac{8}{\sqrt{29}}$

51. $\vec{r} \cdot \left(\sqrt{2}\hat{i} + \hat{j} + \hat{k}\right) = 2$

52. $\hat{i} + 2\hat{j} + 3\hat{k}, \sqrt{14}$

54. $7x + 13y + 4z = 9, \cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$.

CHAPTER 12

LINEAR PROGRAMMING

POINTS TO REMEMBER

- Linear programming is the process used to obtain minimum or maximum value of the linear objective function under known linear constraints.
- **Objective Functions :** Linear function $z = ax + by$ where a and b are constants, which has to be maximized or minimized is called a linear objective function.
- **Constraints :** The linear inequalities or inequations or restrictions on the variables of a linear programming problem.
- **Feasible Region :** It is defined as a set of points which satisfy all the constraints.
- **To Find Feasible Region :** Draw the graph of all the linear inequations and shade common region determined by all the constraints.
- **Feasible Solutions :** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- **Optimal Feasible Solution :** Feasible solution which optimizes the objective function is called optimal feasible solution.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Solve the following L.P.P. graphically

Minimise and maximise $\quad z = 3x + 9y$

Subject to the constraints $\quad x + 3y \le 60$

$$x + y \ge 10$$

$$x \le y$$

$$x \ge 0,\ y \ge 0$$

2. Determine graphically the minimum value of the objective function $z = -50x + 20y$, subject to the constraints

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0,\ y \geq 0$$

3. Two tailors *A* and *B* earn Rs. 150 and *Rs*. 200 per day respectively. *A* can stitch 6 shirts and 4 pants per day, while *B* can stitch 10 shirts and 4 pants per day. Formulate the above L.P.P. mathematically and hence solve it to minimise the labour cost to produce at least 60 shirts and 32 pants.

4. There are two types of fertilisers *A* and *B*. *A* consists of 10% nitrogen and 6% phosphoric acid and *B* consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If *A* costs Rs. 61 kg and *B* costs Rs. 51 kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?

5. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2. Market price of one share of S_1 is Rs 180 and S_2 is Rs. 120. He wishes to purchase a maximum of ten shares only. If one share of type S_1 gives a yield of Rs. 11 and of type S_2 yields Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

6. A company manufactures two types of lamps say *A* and *B*. Both lamps go through a cutter and then a finisher. Lamp *A* requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp *B* requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp *B* is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan

at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

8. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as L.P.P. and then solve it graphically.

9. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and one unit of capital is required. If X and Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

10. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows:

Machine	***Area Occupied***	***Labour Force***	***Daily Output (In units)***
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output.

11. A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below :

Types of Cup	***Machine***		
	I	***II***	***III***
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise and on B is 50 paise, find how many cups of each type should be manufactured to maximise the profit per day.

12. A company produces two types of belts *A* and *B*. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type *A* requires twice as much time as belt of type *B*. The company can produce almost 1000 belts of type *B* per day. Material for 800 belts per day is available. Almost 400 buckles for belts of type *A* and 700 for type *B* are available per day. How much belts of each type should the company produce so as to maximize the profit?

13. Two Godowns *X* and *Y* have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shop *A*, *B* and *C* whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintals from the godowns to the shops are given in following table :

To / ***From***	***Cost of transportation (in Rs. per quintal)***	
	X	***Y***
A	6.00	4.00
B	3.00	2.00
C	2.50	3.00

How should the supplies be transported to ration shops from godowns to minimize the transportation cost?

14. An Aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However atleast four times as many passengers prefer to travel by second class than by first class. Determine, how many tickets of each type must be sold to maximize profit for the airline.

15. A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods *A* and *B* are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food *A* contains 200 unit of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food *B* contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food *A* and *B* should be used to have least cost but it must satisfy the requirements of the sick person.

ANSWERS

1. Min $z = 60$ at $x = 5$, $y = 5$.

   Max $z = 180$ at the two corner points (0, 20) and (15, 15).

2. No minimum value.

3. Minimum cost = Rs. 1350 at 5 days of *A* and 3 days of *B*.

4. 100 kg. of fertiliser *A* and 80 kg of fertilisers *B*; minimum cost *Rs.* 1000.

5. Maximum Profit = Rs. 95 with 5 shares of each type.

6. Lamps of type $A = 40$, Lamps of type $B = 20$.

7. Fan : 8; Sewing machine : 12, Max. Profit = Rs. 392.

8. At 25 km/h he should travel 50/3 km, At 40 km/h, 40/3 km. Max. distance 30 km in 1 hr.

9. *X* : 2 units; *Y* : 6 units; Maximum revenue Rs. 760.

10. Type *A* : 6; Type *B* : 0

11. Cup *A* : 15; Cup *B* : 30

12. Maximum profit Rs. 1300, No. of belts of type $A = 200$ No. of bells of type $B = 600$.

13. From *X* to *A*, *B* and *C* : 10 quintals, 50 quintals and 40 quintals respectively.

    From *Y* to *A, B, C* : 50 quintals, NIL and NIL respectively.

14. No. of first class tickets = 40, No. of 2nd class tickets = 160.

15. Food *A* : 5 units, Food *B* : 30 units.

CHAPTER 13

PROBABILITY

POINTS TO REMEMBER

- **Conditional Probability :** If A and B are two events associated with any random experiment, then $P(A/B)$ represents the probability of occurrence of event-A knowing that event B has already occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$$

$P(B) \neq 0$, means that the event should not be impossible.

$$P(A \cap B) = P(A \text{ and } B) = P(B) \times P(A/B)$$

Similarly $\quad P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/AB)$

- **Multiplication Theorem on Probability :** If the events A and B are associated with any random experiment and the occurrence of one depends on the other then

$$P(A \cap B) = P(A) \times P(B/A) \text{ where } P(A) \neq 0$$

- When the occurrence of one does not depend on the other then these events are said to be independent events.

Here $\quad P(A/B) = P(A)$ and $P(B/A) = P(B)$

$$P(A \cap B) = P(A) \times P(B)$$

- Theorem on total probability : If E_1, E_2, E_3..., E_n be a partition of sample space and E_1, E_2... E_n all have non-zero probability. A be any event associated with sample space S, which occurs with E_1 or E_2,...... or E_n, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + ... + P(E_n) \cdot P(A/E_n).$$

- **Bayes' theorem :** Let S be the sample space and E_1, E_2 ... E_n be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1, or E_2 or ... E_n, then.

$$P\left(E_i/A\right) = \frac{P\left(E_i\right)P\left(A/E_i\right)}{\sum_{i=1}^{n} P\left(E_i\right)P\left(A/E_i\right)}$$

- **Random variable :** It is real valued function whose domain is the sample space of random experiment.

- **Probability distribution :** It is a system of number of random variable (X), such that

X:	x_1	x_2	x_3...	x_n
$P(X)$:	$P(x_1)$	$P(x_2)$	$P(x_3)$...	$P(x_n)$

Where $P\left(x_i\right) > 0$ and $\sum_{i=1}^{n} P\left(x_i\right) = 1$

- Mean or expectation of a random variables (X) is denoted by $E(X)$

$$E\left(X\right) = \mu = \sum_{i=1}^{n} x_i \; P\left(x_i\right)$$

- Variance of X denoted by var(X) or σ_x^2 and

$$\text{var}(X) = \sigma_x^2 = \sum_{i=1}^{n} \left(x_i - \mu\right)^2 P\left(x_i\right)$$

- The non-negative number $\sigma_x = \sqrt{\text{var}\left(X\right)}$ is called standard deviation of random variable X.

- **Bernoulli Trials :** Trials of random experiment are called Bernoulli trials if:

(i) Number of trials is finite.

(ii) Trials are independent.

(iii) Each trial has exactly two outcomes-either success or failure.

(iv) Probability of success remains same in each trial.

- **Binomial Distribution :**

  $P(X = r) = {}^nC_r\, q^{n-r}\, p^r$, where $r = 0, 1, 2, \ldots n$

  p = Probability of Success

  q = Probability of Failure

  n = total number of trails

  r = value of random variable.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find $P(A/B)$ if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$
2. Find $P(A \cap B)$ if A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$
3. A soldier fires three bullets on enemy. The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive?
4. What is the probability that a leap year has 53 Sundays?
5. 20 cards are numbered 1 to 20. One card is drawn at random. What is the probability that the number on the card will be a multiple of 4?
6. Three coins are tossed once. Find the probability of getting at least one head.
7. The probability that a student is not a swimmer is $\frac{1}{5}$. Find the probability that out of 5 students, 4 are swimmers.
8. Find $P(A/B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$
9. A random variable X has the following probability distribution.

X	**0**	**1**	**2**	**3**	**4**	**5**
$P(X)$	$\frac{1}{15}$	k	$\frac{15k-2}{15}$	k	$\frac{15k-1}{15}$	$\frac{1}{15}$

   Find the value of k.

10. A random variable X, taking values 0, 1, 2 has the following probability distribution for some number k.

$$P(X) = \begin{cases} k & \text{if } X = 0 \\ 2k & \text{if } X = 1 \\ 3k & \text{if } X = 2 \end{cases}, \text{ find } k.$$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved.

12. A die is rolled. If the outcome is an even number, what is the probability that it is a prime?

13. If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find P (not A and not B).

14. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or of 7.

15. A can hit a target 4 times in 5 shots B three times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that atleast two shots hit.

16. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

17. A and B throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.

18. If A and B are events such that $P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5}$ and $P(B) = p$ find p if events

    (i) are mutually exclusive,

    (ii) are independent.

19. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

20. Two cards are drawn from a pack of well shuffled 52 cards one by one with replacement. Getting an ace or a spade is considered a success. Find the probability distribution for the number of successes.

21. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/looses.

22. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females.

23. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?

24. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

25. Find the variance of the number obtained on a throw of an unbiased die.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

26. In a hurdle race, a player has to cross 8 hurdles. The probability that he will clear a hurdle is $\frac{4}{5}$, what is the probability that he will knock down in fewer than 2 hurdles?

27. Bag A contains 4 red, 3 white and 2 black balls. Bag B contains 3 red, 2 white and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red. Find the probability that the transferred ball is black.

28. If a fair coin is tossed 10 times, find the probability of getting.

    (i) exactly six heads, (ii) at least six heads,

    (iii) at most six heads.

29. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter by other means of transport are respectively $\frac{3}{13}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus and scooter respectively but if comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

30. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six.

31. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively one of the insured persons meets with an accident. What is the probability that he is a scooter driver?

32. Two cards from a pack of 52 cards are lost. One card is drawn from the remaining cards. If drawn card is heart, find the probability that the lost cards were both hearts.

33. A box *X* contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag *Y*.

34. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, given that he answered correctly?

35. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the die?

36. In a bolt factory machines *A, B* and *C* manufacture 60%, 30% and 10% of the total bolts respectively, 2%, 5% and 10% of the bolts produced by

them respectively are defective. A bolt is picked up at random from the product and is found to be defective. What is the probability that it has been manufactured by machine A?

37. Two urns A and B contain 6 black and 4 white, 4 black and 6 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

38. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

39. Write the probability distribution for the number of heads obtained when three coins are tossed together. Also, find the mean and variance of the number of heads.

40. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

ANSWERS

1. 0.3
2. $\frac{3}{10}$
3. $(0.\ 3)^3$
4. $\frac{2}{7}$
5. $\frac{1}{4}$
6. $\frac{7}{8}$
7. $\left(\frac{4}{5}\right)^4$
8. $\frac{16}{25}$
9. $k = \frac{1}{5}$
10. $k = \frac{1}{6}$
11. $\frac{3}{4}$
12. $\frac{1}{3}$
13. $\frac{3}{8}$
14. $\frac{8}{25}$

15. $\frac{5}{6}$

16. $\frac{5}{9}$

17. $\frac{6}{11}, \frac{5}{11}$

18. (i) $p = \frac{1}{10}$, (ii) $p = \frac{1}{5}$

19. 0.3678

20.

X	**0**	**1**	**2**
P(*X*)	81/169	72/169	16/169

21. $-\frac{91}{54}$

22. $\frac{3}{4}$

23. $\frac{1}{17}$

24. $1 - \left(\frac{9}{10}\right)^{10}$

25. $\text{var}\,(X) = \frac{35}{12}$.

26. $\frac{12}{5}\left(\frac{4}{5}\right)^{7}$.

27. $\frac{2}{33}$

28. (i) $\frac{105}{512}$ (ii) $\frac{193}{512}$ (iii) $\frac{53}{64}$

29. $\frac{1}{2}$

30. $\frac{3}{8}$

31. $\frac{1}{52}$

32. $\frac{22}{425}$

33. $\frac{25}{52}$

34. $\frac{12}{13}$

35. $\frac{8}{11}$

36. $\frac{12}{37}$

37. $\frac{5}{7}$

38. Mean $= \frac{8}{13}$, Variance $= \frac{1200}{287}$

39.

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean $= \frac{3}{2}$

Variance $= \frac{3}{4}$

40. $\frac{2}{9}$

MODEL PAPER - I

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 100***

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and *C.* Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 carry one mark each.

1. Find the value of x, if

$$\begin{pmatrix} 5x + y & -y \\ 2y - x & 3 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -3 & 3 \end{pmatrix}$$

2. Let * be a binary operation on N given by $a * b$ = HCF (a, b), $a, b \in N$. Write the value of 6 * 4.

3. Evaluate : $\displaystyle\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1 - x^2}}\, dx$

4. Evaluate : $\int \frac{\sec^2 (\log x)}{x} dx$

5. Write the principal value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

6. Write the value of the determinant :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

7. Find the value of x from the following :

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$$

8. Find the value of $\left(\hat{i} \times \hat{j}\right) \cdot \hat{k} + \left(\hat{j} \times \hat{k}\right) \cdot \hat{i} + \left(\hat{k} \times \hat{i}\right) \cdot \hat{j}$

9. Write the direction cosines of the line equally inclined to the three coordinate axes.

10. If $\overrightarrow{p}$ is a unit vector and $\left(\overrightarrow{x} - \overrightarrow{p}\right).\left(\overrightarrow{x} + \overrightarrow{p}\right) = 80$, then find $\left|\overrightarrow{x}\right|$.

SECTION B

Question numbers 11 to 22 carry 4 marks each.

11. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of (*a*) the perimeter, (*b*) the area of the rectangle.

OR

Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

12. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

13. Consider $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ defined as $f(x) = \frac{4x}{3x + 4}$

Show that f is invertible. Hence find f^{-1}.

14. Evaluate : $\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$.

OR

Evaluate : $\int x \sin^{-1} x \, dx$.

15. If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

16. In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

17. Using properties of determinants, prove the following :

$$\begin{vmatrix} a + b + c & -c & -b \\ -c & a + b + c & -a \\ -b & -a & a + b + c \end{vmatrix} = 2(a + b)(b + c)(c + a)$$

18. Solve the following differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right).$$

19. Solve the following differential equation :

$$\cos^2 x \cdot \frac{dy}{dx} + y = \tan x.$$

20. Find the shortest distance between the lines

$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$$

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

21. Prove the following :

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right).$$

OR

Solve for x :

$$2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$$

22. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

OR

$\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ are three coplanar vectors. Show that $\left(\overrightarrow{a}+\overrightarrow{b}\right)$, $\left(\overrightarrow{b}+\overrightarrow{c}\right)$ and $\left(\overrightarrow{c}+\overrightarrow{a}\right)$ are also coplanar.

SECTION C

Question number 23 to 29 carry 6 marks each.

23. Find the equation of the plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6). Also find the distance of the point P(6, 5, 9) from the plane.

24. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

25. Evaluate : $\displaystyle\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$.

26. Using matrices, solve the following system of equation :

$$x + y + z = 6$$

$$x + 2z = 7$$

$$3x + y + z = 12$$

OR

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$

27. Coloured balls are distributed in three bags as shown in the following table :

Bag	*Colour of the Ball*		
	Red	***White***	***Black***
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I?

28. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that be can sell all the items that he can buy, how should he invest his money in order to maximise the profit? Formulate this as a linear programming problem and solve it graphically.

29. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

OR

A tank with rectangular base and rectangular sides open at the top is to be constructed so that it's depts is 2m and volume is $8m^3$. If building of tank cost Rs. 70 per *sq* meter for the base and Rs. 45 per *sq* meter for the sides. What is the cost of least expensive tank.

MODEL PAPER - I

SOLUTIONS AND MARKING SCHEME

SECTION A

Note : For 1 mark questions in Section A, full marks are given if answer is correct (i.e. the last step of the solution). Here, solution is given for your help.

Marks

1. We are given

$$\begin{bmatrix} 5x + y & -y \\ 2y - x & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -5 & 3 \end{bmatrix}$$

$\therefore \quad 5x + y = 4$ and $-y = 1$

$\therefore \quad y = -1$ and $5x - 1 = 4$

or $\quad 5x = 5$

$\therefore \quad x = 1$ ...(1)

2. $6 * 4$ = HCF of 6 and 4 = 2. ...(1)

3. $$\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1 - x^2}}\, dx = \left|\sin^{-1} x\right|_0^{1/\sqrt{2}}$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$ ...(1)

4. Let $\quad I = \int \frac{\sec^2 (\log x)}{x}\, dx$

Let $\quad \log x = t$

then $\quad \frac{1}{x} dx = dt$

Marks

or $\quad dx = x\ dt$

$\therefore \quad I = \int \sec^2 t\ dt$

$= \tan t + c$

$= \tan(\log x) + c$ ...(1)

5. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{5\pi}{6}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right]$

$= \frac{5\pi}{6}$ ...(1)

6. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$

$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

$= 0$ ...(1)

7. Here $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

or $\quad 2x^2 - 8 = 0$

or $\quad x^2 - 4 = 0$

$x = \pm 2$ ...(1)

8. $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{k} \times \hat{i}) \cdot \hat{j}$

$= \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j}$

$= 1 + 1 + 1$

$= 3$ ...(1)

Marks

9. The d.c. of a line equally inclined to the coordinate axes are $\left(\frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}\right)$. ...(1)

10. $$(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$$

$$\therefore \quad |\vec{x}|^2 - |\vec{p}|^2 = 80$$

As $\vec{p}$ is a unit vector,

$$|\vec{p}| = 1$$

$$\therefore \quad |\vec{x}|^2 - 1 = 80$$

$$\text{or} \quad |\vec{x}|^2 = 81$$

$$\therefore \quad |x| = 9$$ ...(1)

SECTION B

11. Let P be the perimeter and A be the area of the rectangle at any time t, then

$$P = 2(x + y) \text{ and } A = xy$$

It is given that $\frac{dx}{dt} = -5$ cm/minute

and $\frac{dy}{dt} = 4$ cm/minute ...(1)

(i) We have $P = 2(x + y)$

$$\therefore \quad \frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

Marks

$= 2\ (-5 + 4)$

$= -\ 2$ cm/minute ...(1½)

(ii) We have $A = xy$

$\therefore \quad \dfrac{dA}{dt} = x\dfrac{dy}{dt} + y\dfrac{dx}{dt}$

$= [8 \times 4 + 6\ (-5)]$ ......($\because$ $x = 8$ and $y = 6$)

$= (32 - 30)$

$= 2$ cm^2/minute ...(1½)

OR

The given function is

$$f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$$

$$\therefore \quad f'(x) = \cos x - \sin x$$

$$= -\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$$

$$= -\sqrt{2}\left(\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}\right)$$

$$= -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) \qquad ...(1)$$

For strictly decreasing function,

$$f'(x) < 0$$

$$\therefore \quad -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) < 0$$

or $$\sin\left(x - \frac{\pi}{4}\right) > 0$$

or $$0 < x - \frac{\pi}{4} < \pi$$

or $$\frac{\pi}{4} < x < \pi + \frac{\pi}{4}$$

or $$\frac{\pi}{4} < x < \frac{5\pi}{4}$$

Thus $f(x)$ is a strictly decreasing function on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ ...(2)

As sin x and cos x are well defined in $[0, 2\pi]$,

$f(x) = \sin x + \cos x$ is an increasing function in the complement of interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

i.e., in $$\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$$ ...(1)

12. We are given

$$(\cos x)^y = (\sin y)^x$$

Taking log of both sides, we get

$$y \log \cos x = x \log \sin y$$ ...(½)

Differentiating w.r.t. x, we get

$$y \cdot \frac{1}{\cos x} \cdot (-\sin x) + \log \cos x \cdot \frac{dy}{dx} = x \cdot \frac{1}{\sin y} \cdot (\cos y)\frac{dy}{dx} + \log \sin y \cdot 1$$ ...(2)

or $$-y \tan x + \log \cos x \frac{dy}{dx} = x \cot y \frac{dy}{dx} + \log \sin y$$

$$\Rightarrow \quad \frac{dy}{dx}(\log \cos x - x \cot y) = \log \sin y + y \tan x$$ ...(1)

$$\therefore \quad \frac{dy}{dx} = \frac{\log \sin y + y \tan x}{\log \cos x - x \cot y}$$ ...(½)

Marks

13. For showing f is one-one ...(1½)

For showing f is onto ...(1)

As f is one-one and onto f is invertible ...(½)

For finding $f^{-1}(x) = \dfrac{4x}{4-3x}$ ...(1)

14. Let $I = \displaystyle\int \frac{dx}{\sqrt{5-4x-2x^2}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - 2x - x^2}} \quad ...(½)$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - \left(2x + x^2 + 1 - 1\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{7}{2} - (x+1)^2}} \quad ...(1½)$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{7}{2}\right)^2 - (x+1)^2}}$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\frac{\sqrt{7}}{\sqrt{2}}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right) + c \quad ...(2)$$

Marks

OR

Let $\quad I = \int x \sin^{-1} x \; dx$

$$= \int \sin^{-1} x . x \; dx$$

$$= \sin^{-1} x \cdot \frac{x^2}{2} - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \qquad ...(1)$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}} \qquad ...(1)$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \cdot \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right] - \frac{1}{2} \sin^{-1} x + c$$

...(1)

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + c$$

$$= \frac{x^2 \sin^{-1} x}{2} - \frac{1}{4} \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + c \qquad ...(1)$$

15. We have

$$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

Marks

$\Rightarrow \qquad y\sqrt{1-x^2} = \sin^{-1} x$

Differentiating w.r.t. x, we get

$$y.\frac{(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

or $\qquad -xy + \left(1-x^2\right)\frac{dy}{dx} = 1 \qquad$ ...(1½)

Differentiating again,

$$-x\frac{dy}{dx} - y + \left(1-x^2\right)\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) = 0$$

or $\qquad \left(1-x^2\right)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0 \qquad$ ...(2½)

which is the required result.

16. Here $p = \frac{1}{3}$, $q = 1 - \frac{1}{3} = \frac{2}{3}$, and $n = 5$

Let x denote the number of correct answers. ...(1)

Probability of r successes is given by

$$P(X = r) = {}^nC_r p^r q^{n-r},\ r = 1, 2, 3.....$$

$\therefore \qquad P(X = 4 \text{ or } 5) = P(X = 4) + P(X = 5) \qquad$ ...(½)

$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5 \qquad \text{...(1)}$$

$$= 5\cdot\frac{2}{3^5} + 1\cdot\frac{1}{3^5}$$

$$= \frac{10}{243} + \frac{1}{243} = \frac{11}{243}. \qquad \text{...(1½)}$$

Marks

17. LHS $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$R_1 \to R_1 + R_2$

$R_2 \to R_2 + R_3$

$$\begin{vmatrix} a+b & a+b & -(a+b) \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix} \qquad \text{...(2)}$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ b & -a & a+b+c \end{vmatrix} \qquad \text{...(½)}$$

$C_1 \to C_1 + C_3$

$$= (a+b)(b+c)\begin{vmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ c+a & -a & a+b+c \end{vmatrix} \qquad \text{...(1)}$$

$= 2(a+b)\ (b+c)\ (c+a)$ ...(½)

18. The given differential equation is

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

or $$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

Let $$y = zx$$

$\therefore$ $$\frac{dy}{dx} = z + x\frac{dz}{dx} \qquad \text{...(1)}$$

Marks

$$\therefore \qquad z + x\frac{dz}{dx} = z - \tan z$$

or $$x\frac{dz}{dx} = -\tan z \qquad ...(1)$$

or $$\int \cot z \; dz + \int \frac{dx}{x} = 0 \qquad ...(½)$$

$$\therefore \qquad \log \sin z + \log x = \log c$$

or $$\log (x \sin z) = \log c \qquad ...(1)$$

or $$x \sin\left(\frac{y}{x}\right) = c \qquad ...(½)$$

which is the required solution.

19. The given differential equation is

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

or $$\frac{dy}{dx} + \sec^2 x. \; y = \tan x. \sec^2 x$$

It is a linear differential equation

$$\text{Integrating factor} = e^{\int \sec^2 x \, dx} = e^{\tan x} \qquad ...(1)$$

$\therefore$ Solution of the differential equation is

$$y.e^{\tan x} = \int e^{\tan x}. \tan x \sec^2 x \, dx + c \qquad ...(½)$$

Now, we find $$I_1 = \int e^{\tan x} \cdot \tan x \sec^2 x \, dx$$

Let $$\tan x = t, \; \sec^2 x \; dx = dt$$

Marks

$\therefore \quad I_1 = \int te^t \, dt$

$$= t.e^t - \int e^t dt$$

$$= t \, . \, e^t - e^t$$

$$= (t - 1)e^t = (\tan x - 1) \, e^{\tan x} \quad ...(2)$$

$\therefore$ From (i), solution is

$$y \, . \, e^{\tan x} = (\tan x - 1) \, e^{\tan x} + c$$

or $\quad y = (\tan x - 1) + ce^{-\tan x} \quad ...(½)$

20. Equations of the two lines are :

$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$$

or $\quad \vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right) \quad ...(i)$

and $\quad \vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right) \quad ...(ii)$

Here $\quad \vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

and $\quad \vec{b_1} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k} \quad ...(1)$

$\therefore \quad \vec{a_2} - \vec{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)$

$$= \hat{i} - 3\hat{j} - 2\hat{k} \quad ...(½)$$

and $\quad \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{i} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$$= \hat{i}(-3) - \hat{j}(0) + \hat{k}(3)$$

$$= -3\hat{i} + 3\hat{k} \quad ...(1)$$

Marks

$\therefore \quad \left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{9+9} = 3\sqrt{2}$

$\therefore \quad$ S.D. between the lines $= \dfrac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}$ ...(½)

$$= \frac{\left|\left(\hat{i} - 3\hat{j} - 2\hat{k}\right) \cdot \left(-3\hat{i} + 3\hat{k}\right)\right|}{3\sqrt{2}}$$

$$= \frac{|-3-6|}{3\sqrt{2}}$$

$$= \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units} \quad \text{...(1)}$$

21. $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right] \quad \text{...(1)}$$

$$\text{...}\left[\because x \in \left(0, \frac{\pi}{4}\right)\right]$$

$$= \cot^{-1}\left[\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) - \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right] \quad \text{...(1)}$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right) \quad \text{...(1)}$$

$$= \cot^{-1}\left[\cot\left(\frac{x}{2}\right)\right]$$

$$= \frac{x}{2} \quad \text{...(1)}$$

OR

The given equation is

$$2 \tan^{-1} (\cos x) = \tan^{-1} (2 \operatorname{cosec} x)$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x) \qquad ...(1½)$$

$$\Rightarrow \quad \frac{2\cos x}{\sin^2 x} = 2 \operatorname{cosec} x \qquad ...(1)$$

$$\Rightarrow \quad \cos x = \operatorname{cosec} x \,.\, \sin^2 x$$

$$\Rightarrow \quad \cos x = \sin x$$

$$\therefore \quad x = \frac{\pi}{4} \qquad ...(1½)$$

22. Unit vector along the sum of vectors

$$\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k} \text{ and } \vec{b} = \lambda\hat{i} + 2\hat{j} + 3\hat{k} \text{ is}$$

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + (-2)^2}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \qquad ...(1½)$$

We are given that dot product of above unit vector with the vector $\hat{i} + \hat{j} + \hat{k}$ is 1.

$$\therefore \quad \frac{(2+\lambda)}{\sqrt{\lambda^2 + 4\lambda + 44}} \cdot 1 + \frac{6}{\sqrt{\lambda^2 + 4\lambda + 44}} - \frac{2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1 \qquad ...(1)$$

or $\quad 2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44}$

or $\quad (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$

or $\quad \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

Marks

or $8\lambda = 8$

or $\lambda = 1$ ...(1½)

OR

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar $\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$ ...(½)

$\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar if

$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 0$ (½)

For showing

$$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}] \quad (3)$$

SECTION C

23. Equation of the plane through the points A (3, – 1, 2), B (5, 2, 4) and C (–1, –1, 6) is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \quad ...(2½)$$

i.e. $3x - 4y + 3z = 19$ ...(1½)

Distance of point (6, 5, 9) from plane $3x - 4y + 3z = 19$

$$= \frac{|18 - 20 + 27 - 19|}{\sqrt{9 + 16 + 9}}$$

$$= \frac{6}{\sqrt{34}} \text{ units} \quad ...(1)$$

24. The given parabola is $y^2 = x$ ....(i)

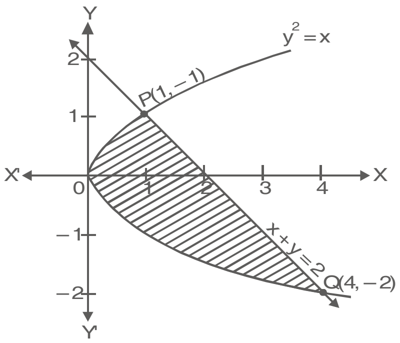
It represents a parabola with vertex at O (0, 0)

Marks

The given line is

$$x + y = 2$$

or $$x = 2 - y \qquad \text{....(ii)}$$



...(1)

Solving (*i*) and (*ii*), we get the point of intersection P (1, 1) and Q (4, – 2) ...(1)

Required area = Area of the shaded region

$$= \int_{-2}^{1} \left[(2 - y) - y^2\right] dy \qquad \text{...(2)}$$

$$= \left(2y - \frac{y^2}{2} - \frac{y^3}{3}\right)_{-2}^{1} \qquad \text{...(1)}$$

$$= \left[\left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)\right]$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 - \frac{8}{3}\right)$$

$$= \frac{12 - 3 - 2 + 24 + 12 - 16}{6}$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ sq. units} \qquad \text{...(1)}$$

Marks

25. Let $\quad I = \int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

or $\quad I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)}$

or $\quad I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ ....(ii)

Adding (*i*) and (*ii*), we get

$$2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad \text{....(iii)} \qquad \text{...(1)}$$

or $\quad 2I = \pi.2 \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

Using property $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$, If $f(2a - x) = f(x)$ ...(1)

or $\quad I = \pi \int_0^{\pi/2} \frac{\sec^2 x \, dx}{a^2 + b^2 \tan^2 x}$ ...(1)

Let $\tan x = t$ then $\sec^2 x \, dx = dt$

When $x = 0$, $t = 0$ and when $x \to \frac{\pi}{2}, t \to \infty$

$\therefore \quad I = \pi \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2}$ ...(1)

$$= \frac{\pi}{b^2} \int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$$

Marks

$$= \frac{\pi}{b^2} \cdot \frac{1}{a/b}\left[\tan^{-1}\frac{t}{a/b}\right]_0^{\infty}$$ ...(1)

$$= \frac{\pi}{ab}\left[\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$$

$$= \frac{\pi}{ab}\left[\frac{\pi}{2}\right]$$

$$= \frac{\pi^2}{2ab}$$ ...(1)

26. The given system of equations is

$$x + y + z = 6$$

$$x + 2z = 7$$

$$3x + y = z = 12$$

or $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

or $X = A^{-1}B$ ...(i) ...(1)

Now $|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

$= 1(-2) - 1(-5) + 1(1)$

$= -2 + 5 + 1$

$= 4 \neq 0 \Rightarrow A^{-1}$ exists. ...(1)

Now cofactors of elements of Matrix A are :

$A_{11} = (-1)^2(-2) = -2$, $A_{12} = (-1)^3(-5) = 5$, $A_{13} = (-1)^4(I) = 1$

Marks

$A_{21} = (-1)^3 (0) = 0,$ $A_{22} = (1)^4 (-2) = -2,$
$A_{23} = (-1)^5 (-2) = 2$

$A_{31} = (-1)^4 (2) = 2,$ $A_{32} = (-1)^5 (1) = -1,$
$A_{33} = (-1)^6 (-1) = -1$

$$\therefore \qquad \text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \qquad ...(2)$$

$$\therefore \qquad A^{-1} = \frac{\text{adj } A}{|A|} \Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \qquad ...(½)$$

$\therefore$ From (i), $X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -12 + & 0 + & 24 \\ 30 - & 14 - & 12 \\ 6 + & 14 - & 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 3,\ y = 1$ and $z = 2$ ...(1½)

OR

26. By using elementary row transformations, we can write

$$A = IA$$

Marks

i.e., $$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(1)

Applying $R_1 \to R_2 - R_2$, we get

$$\begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(1)

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(1)

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(½)

Applying $R_2 \to R_2 - 2R_3$, we get

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(½)

Applying $R_1 \to R_1$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(½)

Marks

Applying $R_3 \rightarrow R_3 - 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & 12 & 9 \end{bmatrix} A \qquad \text{...(½)}$$

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \qquad \text{...(1)}$$

27. Let the events be

E_1 : Bag I is selected

E_2 : Bag II is selected

E_3 : Bag III is selected

and A : a black and a red ball are drawn ...(1)

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3} \qquad \text{...(1)}$$

$$P(A/E_1) = \frac{1 \times 3}{{}^{6}C_2} = \frac{3}{15} = \frac{1}{5}$$

$$P(A/E_2) = \frac{2 \times 1}{{}^{7}C_2} = \frac{2}{21}$$

$$P(A/E_3) = \frac{4 \times 3}{{}^{12}C_2} = \frac{4 \times 3}{66} = \frac{2}{11} \qquad \text{...(1½)}$$

$$\therefore \quad P(E_1/A) = \frac{P(A/E_1)\,.\,P(E_1)}{P(A/E_1)\ P/(E_1) + P(A/E_2)\ P\left(E_2\right) + P(A/E_3).\,P(E_3)} \qquad \text{...(1)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{21} \times \frac{1}{3} \times \frac{2}{11}} \qquad \text{...(½)}$$

Marks

$$= \frac{\frac{1}{15}}{\frac{1}{15} + \frac{2}{63} \times \frac{2}{33}}$$

$$= \frac{\frac{1}{15}}{\frac{551}{3465}}$$

$$= \frac{1}{15} \times \frac{3465}{551} = \frac{231}{551} \qquad ...(1)$$

28. Let us suppose that the dealer buys x fans and y sewing machines,

Thus L.P. problem is

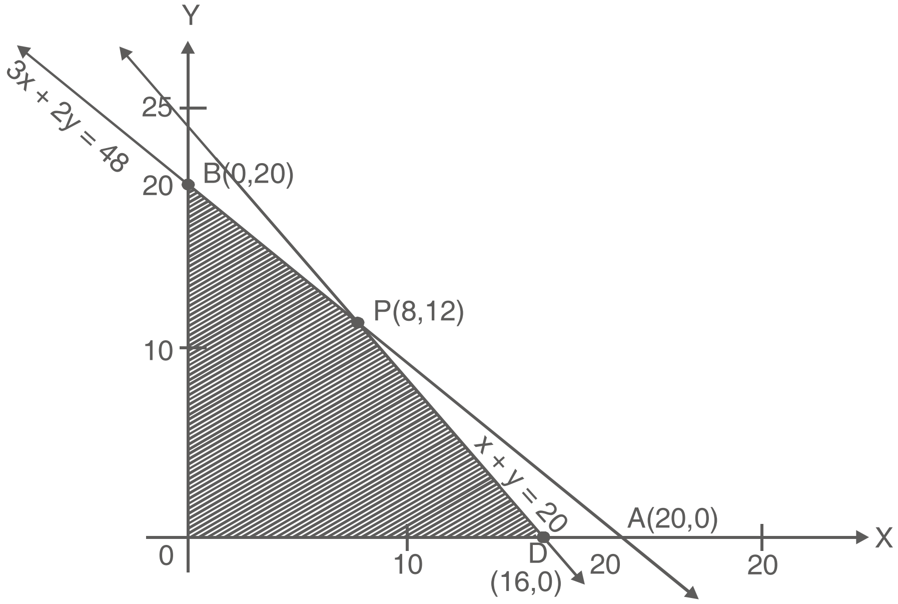
Maximise $\qquad Z = 22x + 18y \qquad ...(1)$

subject to constraints,

$$x + y \le 20$$

$$360x + 240y \le 5760 \text{ or } 3x + 2y \le 48$$

$$x \ge 0,\ y \ge 0 \qquad ...(2)$$



Marks

For correct graph ...(2)

The feasible region ODPB of the L.P.P. is the shaded region which has the corners O (0, 0), D (16, 0), P (8, 12) and B (0, 20)

The values of the objective function Z at O, D, P and B are :

At O, $Z = 22 \times 0 + 18 \times 0 = 0$

At D, $Z = 22 \times 16 + 18 \times 0 = 352$

At P, $Z = 22 \times 8 + 18 \times 12 = 392 \rightarrow$ Maximum

and At B, $Z = 22 \times 0 + 18 \times 20 = 360$

Thus Z is maximum at $x = 8$ and $y = 12$ and the maximum value of z = Rs 392.

Hence the dealer should purchase 8 fans and 12 sewing machines to obtain maximum profit. ...(1)

29. Let ABC be a right angled triangle with base BC = x and hypotenuse AB = y

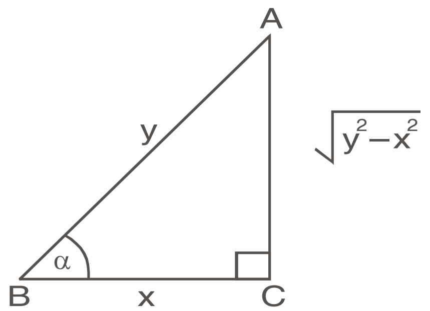
such that

$$x + y = k \text{ where } k \text{ is a constant} \qquad ...(½)$$

Let α be the angle between the base and the hypotenuse.

The area of triangle, $A = \frac{1}{2} BC \times AC$

$$= \frac{1}{2} x \sqrt{y^2 - x^2}$$



...(1½)

Marks

$$\therefore \qquad A^2 = \frac{x^2}{4}(y^2 - x^2)$$

$$= \frac{x^2}{4}\left[(k - x)^2 - x^2\right]$$

or $$A^2 = \frac{x^2}{4}\left[k^2 - 2kx\right] = \frac{k^2x^2 - 2kx^3}{4} \quad ..(i)$$

Differentiating w.r.t. x we get

$$2A\frac{dA}{dx} = \frac{2k^2x - 6kx^2}{4} \quad ....(ii)$$

or $$\frac{dA}{dx} = \frac{k^2x - 3kx^2}{4A} \quad ...(1)$$

For maximum or minimum,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \qquad \frac{k^2x - 3kx^2}{4} = 0$$

$$\Rightarrow \qquad x = \frac{k}{3} \quad ...(1)$$

Differentiating (ii) *w.r.t.x.* we get

$$2\left(\frac{dA}{dx}\right)^2 + 2A\frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4}$$

Putting, $\dfrac{dA}{dx} = 0$ and $x = \dfrac{k}{3}$, we get

$$\frac{d^2A}{dx^2} = \frac{-k^2}{4A} < 0$$

Marks

$\therefore$ A is maximum when $x = \dfrac{k}{3}$ ...(1)

Now $x = \dfrac{k}{3} \Rightarrow y = k - \dfrac{k}{3} = \dfrac{2k}{3}$

$\therefore$ $\cos\alpha = \dfrac{x}{y} \Rightarrow \cos\alpha = \dfrac{k/3}{2k/3} = \dfrac{1}{2}$

$$\alpha = \frac{\pi}{3}$$ ...(1)

OR

30. Let the length of the tank be x metres and breadth by y metres

$\therefore$ Depth of the tank = 2 metre

$\therefore$ Volume = x × y × 2 = 8

$$xy = 4$$

or $y = \dfrac{4}{x}$ ...(1)

Area of base = xy sq m

Area of 4 walls = 2 [2x + 2y] = 4 (x + y)

$\therefore$ Cost C (x ,y) = 70 (xy) + 45 (4x + 4y)

or C (x, y) = 70 × 4 + 180 (x + y) ...(1)

$\therefore$ $C(x) = 280 + 180\left(x + \dfrac{4}{x}\right)$ ...(½)

Now $\dfrac{dC}{dx} = 180\left(1 - \dfrac{4}{x^2}\right)$ ...(1)

For maximum or minimum, $\dfrac{dC}{dx} = 0$

$\therefore$ $180\left(1 - \dfrac{4}{x^2}\right) = 0$

Marks

or $$x^2 = 4$$

or $$x = 2 \quad \text{...(½)}$$

and $$\frac{d^2C}{dx^2} = 180\left(\frac{8}{x^3}\right) > 0$$

$$\left.\frac{d^2C}{dx^2}\right|_{x=2} = 180\left(\frac{8}{8}\right) > 0 \quad \text{...(1)}$$

$\therefore$ C is minimum at $x = 2$

Least Cost = Rs [(280 + 180 (2 + 2)]

= Rs [(280 + 720] = Rs 1000 ...(1)

MODEL PAPER - II

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 100***

General Instructions

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and *C.* Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 carry one mark each.

1. Evaluate : $\int \frac{1}{x + x \log x} dx.$

2. Evaluate : $\int_0^1 \frac{1}{\sqrt{4x + 1}} dx.$

3. If the binary operation * defined on Q, is defined as $a * b = 2a + b - ab$, for all $a, b \in Q,$ find the value of 3 * 4.

4. If $\begin{pmatrix} y + 2x & 5 \\ -x & 3 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix}$, find the value of y.

5. Find a unit vector in the direction of $2\hat{i} - \hat{j} + 2\hat{k}$.

6. Find the direction cosines of the line passing through the following points:

$(-2, 4, -5), (1, 2, 3)$

7. If $A = \left[a_{ij}\right] = \begin{pmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{pmatrix}$ and $B = \left[b_{ij}\right] = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{pmatrix}$, then find $a_{22} + b_{21}$.

8. If $\left|\vec{a}\right| = \sqrt{3}$, $\left|\vec{b}\right| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

9. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, then find the value of k if $|2A| = k\,|A|$.

10. Write the principal value of $\tan^{-1}\left[\tan \frac{3\pi}{4}\right]$.

SECTION B

Question number 11 to 22 carry 4 marks each.

11. Evaluate : $\int \frac{\cos x\, dx}{(2 + \sin x)(3 + 4\sin x)}$.

OR

Evaluate : $\int x^2 \cos^{-1} x \; dx$.

12. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

13. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$.

14. Find the equation of the tangent to the curve $y = \sqrt{4x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

OR

Using differentials, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$.

15. Prove the following :

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right).$$

OR

Solve the following for x :

$$\cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right) + \tan^{-1}\left(\frac{2x}{x^2 - 1}\right) = \frac{2\pi}{3}.$$

16. Find the angle between the line $\frac{x + 1}{2} = \frac{3y + 5}{9} = \frac{3 - z}{-6}$ and the plane $10x + 2y - 11z = 3$.

17. Solve the following differential equation :

$$(x^3 + y^3)\, dy - x^2 y\, dx = 0$$

18. Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x, \ (x \neq 0), \text{ given that } y = 1 \text{ when } x = \frac{\pi}{2}.$$

19. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ba & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

20. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target, what is the probability that

(i) the target is hit?

(ii) exactly one of them hits the target?

21. Find $\frac{dy}{dx}$, if $y^x + x^y = a^b$, where a, b are constants.

22. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are vectors such that $\overrightarrow{a} . \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{c}, \overrightarrow{a} \neq \overrightarrow{0}$ then prove that $\overrightarrow{b} = \overrightarrow{c}$.

SECTION C

Question number 23 to 29 carry 6 marks each.

23. One kind of cake requires 200 g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.

24. Using integration, find the area of the region :

$$\{(x, y) : 9x^2 + y^2 \leq 36 \text{ and } 3x + y \geq 6\}$$

25. Show that the lines $\frac{x+3}{-3} - \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.

26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Show that the total surface area of a closed cuboid with square base and given volume, is minimum, when it is a cube.

27. Using matrices, solve the following system of linear equations :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

28. Evaluate : $\int \frac{x^4 dx}{(x-1)(x^2+1)}$.

OR

Evaluate : $\int_1^4 [|x-1| + |x-2| + |x-4|]\, dx$.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

ANSWERS

MODEL PAPER - II

SECTION A

1. $\log|(1+\log x)| + c$

2. $\frac{1}{2}(\sqrt{5}-1)$

3. -2

4. $y = 3$

5. $\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

6. $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

7. 5

8. $\theta = \frac{\pi}{3}$

9. $k = 4$

10. $-\frac{\pi}{4}$

SECTION B

11. $\frac{1}{5}\log\left|\frac{3+4\sin x}{2+\sin x}\right| + c$

13. **OR** $\frac{1}{at}\sec^3 t$

14. $4x - 2y = 1$

14. **OR** 54.68

15. **OR** $x = 2 - \sqrt{3}$

16. $\theta = \sin^{-1}\left(\frac{8}{21}\right)$

17. $\frac{-x^3}{3y^3} + \log|y| = c$

18. $y\sin x = x + 1 - \frac{\pi}{2}$

20. (i) $\frac{3}{5}$ (ii) $\frac{7}{15}$

21. $\dfrac{dy}{dx} = \dfrac{-\left[y^{x} \log y + yx^{y-1}\right]}{xy^{x-1} + x^{y} \log x}$

SECTION C

23. Maximum number of cakes = 30 of kind one and 10 cakes of another kind.

24. $3(\pi - 2)$ square units

25. $x - 2y + z = 4$.

27. $x = 1, y = 2, z = 3$

28. $\dfrac{x^2}{2} + x + \dfrac{1}{2}\log|x - 1| - \dfrac{1}{4}\log|x^2 + 1| - \dfrac{1}{2}\tan^{-1} x + c$

OR

28. $\dfrac{23}{2}$

29. Mean = 1, Variance = 0.49

RELATIONS AND FUNCTIONS

1 Mark

1) If $f(x) = \frac{x-1}{x+1}, (x \neq 1,-1)$, show that fof^{-1} is an identity function.

2) If $f(x) = Sin^2 x + Sin^2\left(x+\frac{\Pi}{3}\right) + Cosx.Cos\left(x+\frac{\Pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then find the value of $gof(x) = x$

(Ans : 1)

3) Let * be a binary operation on the set of real numbers. If a * b = a+b-ab, 2 * (3 * x) = -7, find the value of x. (Ans : x=-2)

4) Find the number of One-One functions from a finite set A to A, where n(A) = P (Ans : P!)

5) Let A = {4,5,0}. Find the number of binary operations that can be defined on A. (Ans: 3^9)

6) Let $\int:\{R \to R\}$ be a function defined by $f(x) = x^2-3x+4$, for all $x \,\varepsilon\, R$, find the value of $f^{-1}(2)$

Ans: {1,2}

7) Let $f: R \to R$ defined by $f(x) = (ax^2+b)^3$ find the function $g: R \to R$ such that f(g(x)) = g(f(x))

Ans: $\left[\frac{x^{\frac{1}{3}}-b}{a}\right]^{\frac{1}{2}}$

8) If $f(x) = \frac{5x+3}{4x-5}\left(x \neq \frac{5}{4}\right)$, find g(x) such that gof(x) = x Ans: $g(x) = \frac{5x+3}{4x-5}$

9) If $f(x) = \frac{1+x}{1-x}$, show that $f[f(\tan\theta)] = -\cot\theta$

10) Show that $\frac{1}{\sin^3 x} + \cot x + \frac{1}{x^5} + x^3$ is an odd function.

11) Let f, g be two functions defined by

$f(x) = \frac{x}{x+1}, g(x) = \frac{x}{1-x}$, then find $(fog)^{-1}(x)$ Ans: x

12) Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, find the value of α such that f(f(x)) = x Ans: $\alpha = -1$

4 Marks / 6 marks

13) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f(x)+f(y) = f\left(\frac{x+y}{1+xy}\right)$

14) If R is a relation on a set $A(A \neq \phi)$, prove that R is symmetric iff $R^{-1} = R$

15) Show that the relation R on N x N defined by $(a,b)R(c,d) \Leftrightarrow a+d = b+c$ is an equivalence relation.

16) Let $f: R_+ \to [-5,\infty)$ given by $f(x) = 9x^2+6x-5$. Show that f is invertiable with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

17) Show that the relation "congruence modulo 2" on the set Z (set of integers) is an equivalence relation. Also find the equivalence class of 1.

18) If the function $f: R \to A$ given by $f(x) = \dfrac{x^2}{x^2+1}$ is surjection, find the set A.

Ans: A = Range of f(x)=[0,1)

19) Let a relation R on the set R of real numbers be defined as (a,b) $\varepsilon R \Leftrightarrow 1 + ab > 0$ for all a, b ε R. show that R is reflexive and symmetric but not transitive.

20) Let a relation R on the set R of real numbers defined as $(x, y)\varepsilon R \Leftrightarrow x^2 4xy + 3y^2 = 0$. Show that R is reflexive but neither symmetric nor transitive.

21) Let N denote the set of all natural numbers and R be the relation on N x N defined by $(a, b)R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Show that R is an equivalence relation on N x N.

22) Prove that the inverse of an equivalence relation is an equivalence relation.

23) Let $f : A \to B$ be a given function. A relation R in the set A is given by $R = \{(a, b) \; \varepsilon \; A \times A : f(a) = f(b)\}$. Check, if R is an equivalence relation. Ans: Yes

24) Let f and g be real valued functions, such that (fog)(x) = $\cos x^3$ and (gof)(x) = $\cos^3 x$, find the functions f and g. Ans: f(x)=cosx, g(x) = x^3

25) Define a binary operation * on the set A = {0,1,2,3,4,5} as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is an identity element for this operation and each element a of the set is invertible with 6-a being the inverse of a.

26) Show that the function $f : N \to N$ given by f(x) = x - $(-1)^x$ for all $x \; \varepsilon \; N$ is a bijection.

27) Prove that relation R defined on the set N of natural numbers by

x R y $\Leftrightarrow$ $2x^2 - 3xy + y^2 = 0$ is not symmetric but it is reflexive.

28) Let X = {1,2,3,4,5,6,7,8,9}. Let R_1 and R_2 be relations in X given by

R_1 = {(x,y) : x-y is divisible by 3} and

R_2 = {(x,y) ! {x,y} ε {1,4,7} or {x,y} ε {2,5,8}

or {x,y} ε {3,6,9}}. Show that $R_1 = R_2$

29) Determine which of the following functions

f : R $\to$ R are (a) One - One (b) Onto

(i) f(x) = |x| + x

(ii) f(x) = x - [x]

(Ans: (i) and (ii) $\to$ Neither One-One nor Onto)

30) On the set N of natural numbers, define the operation * on N by m*n = gcd(m,n) for all m, n ε N. Show that * is commutative as well as associative.

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INVERSE TRIGONOMETRIC FUNCTIONS

1 Mark

1) Find the value of $\cos\left\{\operatorname{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\Pi}{6}\right\}$ Ans: -1

2) Find the value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$. Ans: Π

3) Solve for x : $\operatorname{Sin}\left[\operatorname{Sin}^{-1}\frac{1}{5}+\operatorname{Cos}^{-1}x\right]=1$ Ans: $\frac{1}{5}$

4) Write the simplest form : $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ Ans: $\frac{\Pi}{4}+\frac{x}{2}$

5) Considering the principal solutions, find the number of solutions of

$\tan^{-1}2x+\tan^{-1}3x=\frac{\Pi}{4}$ Ans: 2

6) Find the principal value of $\operatorname{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\operatorname{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ Ans: $\frac{\Pi}{2}$

7) Find the value of x if $\operatorname{Cosec}^{-1}x+2\operatorname{Cot}^{-1}7+\operatorname{Cos}^{-1}\frac{3}{4}$ Ans: $x=\operatorname{Cosec}^{-1}\frac{125}{117}$

8) If $\operatorname{Cos}^{-1}x = \tan^{-1}x$, show that $\operatorname{Sin}(\cos^{-1}x) = x^2$

9) If x > 0 and $\operatorname{Sin}^{-1}\left(\frac{5}{x}\right)+\operatorname{Sin}^{-1}\left(\frac{12}{x}\right)=\frac{\Pi}{2}$, then find the value of x. Ans: x = 13

10) Prove that $\operatorname{Cos}\left\{2\operatorname{Cot}^{-1}\sqrt{\frac{1-x}{1+x}}\right\}+x=0$

4 Marks / 6 Marks

11) Prove that $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\Pi}{4}$

12) If $x, y, z \,\varepsilon\, [-1,1]$ such that $\operatorname{Sin}^{-1}x+\operatorname{Sin}^{-1}y+\operatorname{Sin}^{-1}z=\frac{3\Pi}{2}$, find the value of

$x^{2006}+y^{2007}+z^{2008}-\frac{9}{x^{2006}+y^{2007}+z^{2008}}$ Ans: zero ; x=1,y=1,z=1

13) If $\operatorname{Cos}^{-1}\frac{x}{a}+\operatorname{Cos}^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$

14) If $\operatorname{Sin}^{-1}x+\operatorname{Sin}^{-1}y+\operatorname{Sin}^{-1}z=\Pi$, prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

15) Prove that : $\operatorname{Sin}2\left[\cot^{-1}\left\{\cos(\tan^{-1}x)\right\}\right]=\sqrt{\frac{x^2+1}{x^2+2}}$

16) In any Triangle ABC, if A = $\tan^{-1}2$ and B = $\tan^{-1}3$, prove that $C=\frac{\Pi}{4}$

17) If $x = \text{cosec}\left[\tan^{-1}\left\{\cos(\cot^{-1}(\sec(\sin^{-1} a)))\right\}\right]$ and

$y = \text{Sec}\left[\cot^{-1}\left\{\sin(\tan^{-1}(\text{cosec}(\cos^{-1} a)))\right\}\right]$ where a $\varepsilon[0,1]$

Find the relationship between x and y in terms of a Ans: $x^2=y^2=3-a^2$

18) Prove that : $\text{Cot}^{-1}\left[\frac{ab+1}{a-b}\right]+\text{Cot}^{-1}\left[\frac{bc+1}{b-c}\right]+\text{Cot}^{-1}\left[\frac{ca+1}{c-a}\right]=0$

19) Solve for x : $\text{Sin}^{-1}\frac{2\alpha}{1+\alpha^2}+\text{Sin}^{-1}\frac{2\beta}{1+\beta^2}=2\tan^{-1}x$ Ans: $x=\frac{\alpha+\beta}{1-\alpha\beta}$

20) Prove : $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left[xy+\sqrt{1-x^2}.\sqrt{1-y^2}\right]$

21) If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c=\Pi$, Prove that a+b+c -= abc

22) Prove that $\tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{zx}{yr}+\tan^{-1}\frac{xy}{zr}=\frac{\Pi}{2}$ where $x^2+y^2+z^2=r^2$

23) Solve for x : $\tan^{-1}(x+1)+\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$ Ans: x = -1

24) Solve : $\text{Sin}\left[2\cos^{-1}\left\{\cot(2\tan^{-1}x)\right\}\right]=0$ Ans: $\pm1, -1\pm\sqrt{2}, 1\pm\sqrt{2}$

25) If $\text{Cos}^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, Prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$

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HIGHLY ORDER THINKING QUESTIONS

HOTS - MATRICES / DETERMINANTS

1) If $a+b+c=0$ and $\begin{vmatrix} a-x & a & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ then prove that either x = 0 or $x=\pm\sqrt{\frac{3}{2}\left(a^2+b^2+c^2\right)}$

2) If $A=\begin{bmatrix} p & q \\ r & -p \end{bmatrix}$ is such that $A^2=I$ then find the value of I-P^2+qr

3) If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A+A^T=I$. Find the possible values of θ $\theta=\Pi/3$

4) Inverse of a square matrix is unique. Give an example to prove it?

5) Prove that $\begin{vmatrix} x-3 & x-4 & x-a \\ x-2 & x-3 & x-b \\ x-1 & x-2 & x-c \end{vmatrix}=0$, where a, b, c are in A.P.

6) Using properties of Determinants prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+ac & c^2 \end{vmatrix}=4a^2b^2c^2$

7) Express $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ as sum of the symmetric and skew symmetric matrices.

8) Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix}=2abc(a+b+c)^2$ (Use properties to prove the above)

9) Prove the determinant $\begin{vmatrix} x & -\sin\alpha & \cos\alpha \\ \sin\alpha & -x & 1 \\ \cos\alpha & 1 & x \end{vmatrix}$ is independent as α (Ans: Scalar term)

10) The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have. 2

11) Find the matrix X such that : $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}X=\begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

12) If f(x) = $3x^2$-9x+7, then for a square matrix A, write f(A) ($3A^2$-9A+7I)

13) Prove that $\begin{bmatrix} (a+1) & (a+2) & (a+2) & 1 \\ (a+2) & (a+3) & a+3) & 1 \\ (a+3) & (a+4) & (a+4) & 1 \end{bmatrix}=-2$

14) If $\begin{bmatrix} \cos^2 A & \cos A \sin A \\ \cos A \sin A & \sin^2 A \end{bmatrix}, Y = \begin{bmatrix} \cos^2 B & \cos B \sin B \\ \cos B \sin B & \sin^2 B \end{bmatrix}$

then show that XY is a zero matrix, provided (A-B) is an odd multiple of $\Pi/2$

15) Give that x = -9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ find the other roots. Hint: Evaluate, find other roots.

16) If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2+xI = yA$. Find A^{-1}

17) If P and Q are equal matrices of same order such that PQ = QP, then prove by induction that $PQ^n = Q^nP$. Further, show that $(PQ)^n = P^n \times Q^n$

18) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 = ?$ I_3

19) If A and B are two matrices such that AB = B and BA = A then A^2+B^2 = _____?______\

20) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \in N$

21) Find the values of a, b, c if the matrix

A = $\begin{vmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{vmatrix}$ satisfy the equation $A^{I}A = I_3$

22) Assume X, Y, Z, W and P are matrices of order (2 x n), (3xk), (n x 3) and (p x k) respectively, then the restriction on n, k and p so that py + my will be defined are : (k=3, p=n)

23) Let A and B be 3 x 3 matrices such that $A^T = -A$ and $B^T = B$. Then the matrix λ (AB+3BA) is skew symmetric matrix for λ. (λ =3)

24) If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A+7I = 0$, use the result to find A^4 $\begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

25) For what value of 'K' the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse. (K = 3/2)

26) If A is a non-singular matrix of order 3 and |A| = -4 Find |adj A| (16)

27) Given A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that $|A| = -10$. Find $a_{11}c_{11} + a_{12}c_{12}$ (10)

28) If $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix} = 0$, then find the value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$ (2)

29) If A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation :

x^2-6x+17 = 0 find A^{-1} $\quad$ $\text{Ans:}\left(\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}\right)$

30) Find the matrix x if $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} x \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

$X = -\frac{1}{4}\begin{bmatrix} -53 & 18 \\ 25 & -10 \end{bmatrix}$

31) If P(a) = $\begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $[P(x)]^{-1} = [P(-x)]$

32) If two matrices A^{-1} and B are given how to find $(AB)^{-1}$ verify with an example.

(Find B^{-1} then find $B^{-1} \times A^{-1}$)

33) If A = $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ Verify $(\text{Adj}A)^{-1} = \text{adj}(A^{-1})$

34) Find the values of a and b such that $A^2 + aI = bA$ where A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ (a=b=8)

35) If $P(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ show that $p(\alpha) \times p(\beta) = p(\alpha+\beta)$

36) If A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ then prove by Mathematical Induction that : $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$

37) If A = $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ find Matrix B such that AB = I, Ans : $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

38) If x, y, z are positive and unequal show that the value of determinant $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is negative.

39) If $A+B+C=\Pi$, show that $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = 0$

40) Find the quadratic function defined by the equation $f(x) = ax^2+bx+c$
if f(o) = 6, f(2) = 11, f(-3) = 6, using determinants.

41) If x, y and z all positive, are p^{th}, q^{th} and r^{th} terms of a G.P. Prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

42) If a, b, c are in A.P. then find the value of : $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$ (O)

43) If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then show that $A^n = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

44) If A = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ find A^2 Hence find A^6 Ans: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

45) Find x, if $[x\ -5\ -1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ Ans: $x = \pm 4\sqrt{3}$

46) If P and Q are invertible matrices of same order, then show that PQ is also invertible.

47) If the points (2,0), (0,5) and (x,y) are collinear give a relation between x and y.

48) Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ find the possible values of x and y, find the values if x = y.

49) If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} a^n & \dfrac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}, n \,\varepsilon\, N$

50) For any square matrix verify A(adj A) = |A|I

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CONTINUITY AND DIFFERENTIABILITY

1 Mark

1) Let f be continuous function on [1,3]. If f takes only rational values for all x and f(2) = 10 then find the value of f(1.5) Ans: 10

2) Let f be a non zero continuous function satisfying f(x+y) = f(x).f(y) for all x, y ε R. If f(2) = 9 then find the value of f(3).

Ans: $3^3=27$, f(x) is of the form a^x.

3) Find $\frac{dy}{dx}$ when $y = x^{x^{x^{\cdots}}}$ Ans: $\frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$

4) Find the set of all points where the function f(x) = 2x|x| is differentiable. Ans: $(-\alpha, \alpha)$

5) If $f^{|}(x) = g(x)$ and $g^{|}(x) = -f(x)$ for all n and $f(2) = f^{|}(2)=4$, find the value of $f^2(24)+g^2(24)$

Ans: 32

6) If $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$ then find $f^{|}(x)$ Ans: $6x^2$

7) Let y = $y = \sqrt{e^{\sqrt{x}}}$, $x > 0$ find $\frac{dy}{dx}$ Ans: $\frac{1}{4}\sqrt{\frac{e^{\sqrt{x}}}{x}}$

8) Verify Rolle's theorem for the function f(x) = sin2x in $\left[0, \frac{\Pi}{2}\right]$ Ans: C= $\frac{\Pi}{4}$

9) Find $\frac{dy}{dx}$ when $y = a^x.x^a$ Ans: $a^x.x^{a-1}(a + x\log a)$

10) Find $\frac{dy}{dx}$ when $y = \tan^{-1}\frac{1+x^2}{1-x^2}$ Ans: $\frac{2x}{1+x^4}$

4 MARKS/6 MARKS

11) Given that $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x < 0 \\ a & , \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}} & , \text{if } x > 0 \end{cases}$

If f(x) is continuous at x = 0, find the value of a. Ans: a=8

12) Show that the function : $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{when } x \neq 0 \\ 0 \;, & \text{when } x = 0 \end{cases}$ is discontinuous at x = 0

13) Is the function $f(x) = \frac{3x+4\tan x}{x}$ continuous at x = 0? If not, how may the function be defined to make it continuous at this point.

Ans: $f(x) = \begin{cases} \frac{3x+4\tan x}{x}, & \text{when } x \neq 0 \\ 7 \;, & \text{when } x = 0 \end{cases}$

14) Find a and b if the function :

$$f(x)=\begin{cases}[1+|\sin x|]^{\frac{a}{|\sin x|}} & , \quad \frac{-\Pi}{6}<x<0\\ 6 & , \quad x=0\\ e^{\frac{\tan 2x}{\tan 3x}} & , \quad 0<x<\frac{\Pi}{6}\end{cases}$$ is continuous on $\left(\frac{-\Pi}{6},\frac{\Pi}{6}\right)$

15) Show that the function : $f(x)=\begin{cases}|x|, & x\le 2\\ [x], & x>2\end{cases}$ is continuous on $[0,2]$

16) Show that Sin|x| is continuous.

17) Show that the function $f(x)=\begin{cases}x+\lambda, & x<1\\ \lambda x^2+1, & x\ge 1\end{cases}$

is continuous function, regardless of the choice of $\lambda \varepsilon R$

18) Determine the values of a, b and c for which the function :

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & , \quad x<0\\ c & , \quad x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}} & , \quad x>0\end{cases}$$ may continuous at x = 0

Ans: $a=-\frac{3}{2}, c=\frac{1}{2}$, b may have any real number.

19) Show that the function f(x) = |Sinx+cosx| is continuous at x = Π

20) The function $f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at x = 0. Find the value of f(x) so that f(x) is continuous at x = 0.

Ans: f(x) to be continuous at x = 0, f(o) = a+b

21) Find all the points of discontinuity of f defined by f(x) = |x|-|x+1| Ans: x=0, -1

22) Let $f(x)=\begin{cases}|x|\cos\frac{1}{x} & \text{if } x\ne 0\\ 0 & \text{if } x=0\end{cases}$ then discuss the continuity of f(x) at x = 0

Ans: Yes, f(x) is continuous at x = 0

23) Discuss the continuity of the following function at x = 0

$$f(x)=\begin{cases}\frac{x^4+x^3+2x^2}{\tan^{-1}x}, & x\ne 0\\ 10 \;, & x=0\end{cases}$$

24) Let $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x} & \text{if } x<\dfrac{\Pi}{2}\\ a & \text{if } x=\dfrac{\Pi}{2}\\ \dfrac{b(1-\sin x)}{(\Pi-2x)^2} & \text{if } x>\dfrac{\Pi}{2}\end{cases}$ If f(x) is continuous at x $x=\dfrac{\Pi}{2}$, find a and b.

Ans : a=½, b = 4

25) Let $\phi(x)$ and $\varphi(x)$ be derivable at x = 3. Show that necessary and sufficient condition for the function defined as :

$f(x)=\begin{cases}\phi(x), & x\le c\\ \varphi(x), & x>c\end{cases}$ to be derivable at $x=c$ are (i) $\phi(c)=\varphi(c)$ (ii) $\phi^{|}(c)=\varphi^{|}(c)$

26) Find $\dfrac{dy}{dx}$ when $y=\mathrm{Sin}^{-1}\left[\dfrac{5x+12\sqrt{1-x^2}}{13}\right]$ Ans: $\dfrac{1}{\sqrt{1-x^2}}$

27) If $y=\mathrm{Sin}^{-1}\left[x^2\sqrt{1-x^2}+x\sqrt{1-x^4}\right]$, prove that $\dfrac{dy}{dx}=\dfrac{2x}{\sqrt{1-x^4}}+\dfrac{1}{\sqrt{1-x^2}}$

28) If $y=\dfrac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{x}{2}\right]$, prove that $\dfrac{dy}{dx}=\dfrac{1}{a+b\cos x}$, $a>b>0$

29) Differentiate w.r.t. x, $y=\tan^{-1}\left[\dfrac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$ Ans: -1

30) Find $\dfrac{dy}{dx}$, when $y=\sin^{-1}\left[x\sqrt{x-1}-\sqrt{x}\sqrt{1-x^2}\right]$ Ans: $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}$

31) Given that $\mathrm{Cos}\dfrac{x}{2}.\mathrm{Cos}\dfrac{x}{4}.\mathrm{Cos}\dfrac{x}{8}\ldots\ldots\ldots=\dfrac{\mathrm{Sin}x}{x}$,

Prove that $\dfrac{1}{2^2}\mathrm{Sec}^2\dfrac{x}{2}+\dfrac{1}{2^4}\mathrm{Sec}^2\dfrac{x}{4}+\ldots\ldots.=\mathrm{Cosec}^2 x-\dfrac{1}{x^2}$

32) Let $y=\mathrm{Cot}^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, find $\dfrac{dy}{dx}$

Ans: $\dfrac{dy}{dx}=\dfrac{1}{2}$, Take $\sqrt{1\pm\sin x}=\cos\dfrac{x}{2}\pm\sin\dfrac{x}{2}$

33) Let $y=\tan^{-1}\left[\dfrac{4x}{1+5x^2}\right]+\tan^{-1}\left[\dfrac{2+3x}{3-2x}\right]$, show that $\dfrac{dy}{dx}=\dfrac{5}{1+25x^2}$

34) Prove that $\dfrac{d}{dx}\left[\dfrac{1}{4\sqrt{2}}\log\left|\dfrac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\dfrac{1}{2\sqrt{2}}\tan^{-1}\dfrac{\sqrt{2}x}{1-x^2}\right]=\dfrac{1}{1+x^4}$

35) If $(x-a)^2 + (y-b)^2 = c^2$ for some c > 0 prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b. Ans: = -c

36) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

37) If $y=\frac{1}{\sqrt{b^2-a^2}}\log\left[\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{x}{2}}\right]$ prove that $\frac{dy}{dx}=\frac{\sec^2\frac{x}{2}}{(b+1)-(b-a)\tan^2\frac{x}{2}}$

38) If $y=|\cos x|+|\sin x|$, find $\frac{dy}{dx}$ at $x=\frac{2\Pi}{3}$

Ans: $\frac{1}{2}\left(\sqrt{3}-1\right)$, Take $y = -\cos x+\sin x$ around $x=\frac{2\Pi}{3}$

39) Find $\frac{dy}{dx}$ when $y=\tan^{-1}\left[\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right]$ Ans: $\frac{1}{3x^{\frac{2}{3}}\left(1+x^{\frac{2}{3}}\right)}$

40) If $y=f\left(\frac{2x-1}{x^2+1}\right)$ and $f^{I}(x)=\sin x^2$, find $\frac{dy}{dx}$ Ans: $\frac{dy}{dx}=\frac{-2x^2+2x+2}{(x^2+1)^2}\operatorname{Sin}\left(\frac{2x-1}{x^2+1}\right)^2$

41) If $y^{\cos x}+(\tan^{-1}x)^y=1$, find $\frac{dy}{dx}$

Ans: $\frac{dy}{dx}=\frac{y^{\cot x}.\operatorname{cosec}^2x.\log y-\left\{\frac{(\tan^{-1}x)^{y-1}.y}{1+x^2}\right\}}{y^{\cot x-1}.\cot x+(\tan^{-1}x)^y.\log(\tan^{-1}x)}$ Use logarithmic differentiation.

42) If $x=\cos\theta+\log\tan\frac{\theta}{2}$, $y=\sin\theta$ find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\Pi}{4}$ Ans: $2\sqrt{2}$

43) Differentiate $\operatorname{Cos}^{-1}\left[\frac{3\cos x-2\sin x}{\sqrt{13}}\right]$ w.r.t. $\operatorname{Sin}^{-1}\left[\frac{5\sin x+4\cos x}{\sqrt{41}}\right]$ Ans: 1

44) If $y=\sqrt{x^2+1}-\log\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$ Ans: $\frac{dy}{dx}=\frac{x^2+\sqrt{1+x^2}+1}{x(1+\sqrt{1+x^2}}$

45) If $x\sqrt{1+y}+y\sqrt{1+x}$, prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

46) Verify Rolle's theorem for the function $f(x)=e^{1-x^2}$ in the interval [-1,1] Ans: C = 0

47) It is given that for the function $f(x) = x^3-6x^2+px+q$ on [1,3]. Rolle's theorem holds with

$C = 2 + \frac{1}{\sqrt{3}}$. Find the values of p and q.

48) If f(x) and g(x) are functions derivable in [a,b] such that f(a) = 4, f(b) = 10, g(a) =1, g(b) =3. Show that for a < c < b, we have $f^{l}(c) = 3g^{l}(c)$.

49) Using LMV Theorem, find a point on the curve $y = (x-3)^2$, where the tangent is parallel to the chord joining (3,0) and (5,4) Ans: (4,1)

50) Verify the Rolle's Theorem for the function f(x) = Sinx-Cosx in the interval $\left[\frac{\Pi}{4}, \frac{5\Pi}{4}\right]$

Ans: $C=\frac{3\Pi}{4}$

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APPLICATION OF DERIVATIVES

1) The slope of the tangent to the curve represented by $x = t^2+3t-8$ and $y = 2t^2-2t-5$ at the point M(2,-1) is

(a) $7/6$ (b) $2/3$ (c) $3/2$ (d) $6/7$ Ans (d)

2) The function $f(x) = 2 \log (x-2) - x^2+4x+1$ increases in the interval.

(a) (1,2) (b) (2,3) (c) $\left(5/2, 3\right)$ (d) (2,4) Ans: (b) and (c)

3) The function $y = \tan^{-1}x - x$ decreases in the interval of

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$ Ans: all

4) The value of a for which the function $f(x) = a\mathrm{Sin}x + \frac{1}{3}\mathrm{Sin}3x$ has an extreme at $x = \Pi/3$ is

(a) 1 (b) -1 (c) (O) (d) 2 Ans: d

5) The co-ordinates of the point p(x,y) in the first quadrant on the ellipse $\frac{x^2}{8}+\frac{y^2}{18}=1$ so that the area of the triangle formed by the tangent at P and the co-ordinate axes is the smallest are given by

(a) (2,3) (b) $\sqrt{8},0$ (d) $\left(\sqrt{18},0\right)$ (d) none of these Ans: (a)

6) The difference between the greatest and the least values of the function

$f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

(a) $2/3$ (b) $8/7$ (c) $9/4$ (d) $3/8$ Ans: (c)

7) If $y = a \log|x| + bx^2+x$ has its extreme values at $x = -1$ and $x = 2$ then

(a) a=2 b= -1 (b) a=2, b = $-½$ (c) a=-2, b=½ (d) none of these Ans: (b)

8) If θ is the semivertical angle of a cone of maximum volume and given slant height, then $\tan\theta$ is given by

(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$ Ans: (c)

9) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$ where $0 \le x \le 1$ then in this interval

(a) both f(x) and g(x) are increasing
(b) both f(x) and g(x) are decreasing
(c) f(x) is an increasing function
(d) g(x) is an increasing function Ans: (c)

10) If $f(x) = \begin{cases} 3x^2+12x-1 & : \ -1 \le x \le 2 \\ 37-x & : \ 2 < x \le 3 \end{cases}$ then

(a) f(x) is increasing on $[-1,2]$
(b) f(x) is continuous on $[-1,3]$
(c) $f^{|}(2)$ doesn't exist
(d) f(x) has the maximum value at x = 2 Ans: (a),(b),(c),(d)

11) The function $\frac{\sin(x+\alpha)}{\sin(x+\beta)}$ has no maximum or minimum value if

(a) $\beta-\alpha = k\Pi$ (b) $\beta - \alpha \ne k\Pi$ (c) $\beta - \alpha = 2k\Pi$
(d) None of these where K is an integer. Ans: (b)

12) If $f(x)=\dfrac{x^2-1}{x^2+1}$ for every real number then minimum value of f

(a) does not exist (b) is not attained even though f is bounded
(c) is equal to 1 (d) is equal to -1 Ans: (d)

13) If the line ax+by+c = 0 is normal to the curve xy = 1 then

(a) a>0, b>0 (b) a>0, b<0 (d) a<0, b>0 (d) a<0, b<0 Ans: (b),(c)

14) The tangent to the curve

$x=a\sqrt{\cos 2\theta}\cos\theta \quad y=a\sqrt{\cos 2\theta}\sin\theta$ at the point corresponding to $\theta=\Pi/6$ is

(a) Parallel to the x-axis (b) Parallel to the y-axis
(c) Parallel to the line y = x (d) none of these Ans: (a)

15) The minimum value of f(x) = |3−x| + |2+x| + |5−x| is

(a) 0 (b) 7 (c) 8 (d) 10 Ans: (b)

16) If $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ then $\dfrac{d^2y}{dx^2}$ at $\theta=\Pi/2$ is

(a) $\dfrac{1}{a}$ (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{a}$ (d) $-\dfrac{1}{2a}$ Ans: (c)

17) If $y=\log\tan\left(\dfrac{\Pi}{4}+\dfrac{\Pi}{2}\right)$ then $\dfrac{dy}{dx}$ is

(a) 0 (b) cosx (c) −secx (d) secx Ans: (d)

18) 'C' on LMV for f(x) = x^2−3x in [0,1] is

(a) 0 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) does not exist Ans: (b)

19) The values of a and b for which the function $f(x)=\begin{cases} ax+1 & x\le 3 \\ bx+3 & x>3 \end{cases}$ is continuous at x = 3 are

(a) 3a + 2b = 5 (b) 3a = 2+3b (c) 3,2 (d) none of these Ans: (b)

20) The tangents to the curve y = x^3+6 at the points (-1,5) and (1,7) are

(a) Perpendicular (b) parallel (c) coincident (d) none of these Ans: (b)

21) If $\dfrac{dy}{dx}=0$ then the tangent is

(a) Parallel to x-axis (b) parallel to y-axis (c) Perpendicular to x-axis
(d) perpendicular to y-axis Ans: (a)

22) If the slope of the tangent is zero at (x_1, y_1) then the equation of the tangent at (x_1,y_1) is

(a) $y_1=mx_1+c$ (b) $y_1= mx_1$ (c) $y-y_1$ (d) y=0 Ans: (c)

23) The function $f(x)=-\dfrac{x}{2}+\sin x$ is always increasing in

(a) $\left(\dfrac{-\Pi}{2},\dfrac{\Pi}{2}\right)$ (b) $\left(0,\dfrac{\Pi}{4}\right)$ (c) $\left(\dfrac{\Pi}{4},\dfrac{\Pi}{2}\right)$ (d) $\left(\dfrac{-\Pi}{3},\dfrac{\Pi}{3}\right)$ Ans: (d)\

24) The least value of 'a' such that the function f(x) = x^2+ax+1 is strictly increasing on (1,2) is

(a) −2 (b) 2 (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$ Ans: (a)

25) The least value of f(x) = tan-1 (sinx + cosx) strictly increasing is

(a) $\left(\frac{\Pi}{4},\frac{\Pi}{2}\right)$ (b) $\left(0,\frac{\Pi}{2}\right)$ (c) 0 (d) none of these Ans: (d)

4-6 Marks

26) Determine the points of maxima and minima of the function

$f(x)=\frac{1}{8}\log x-bx+cx^2$ where $b\geq 0$

Ans: f has maxima at $2=\frac{1}{4}\left(b-\sqrt{b^2-1}\right)$ and minima at $\beta=\frac{1}{4}\left(b+\sqrt{b^2-1}\right)$

27) Find the interval in which the following functions are increasing or decreasing

(a) $y=\log\left(x+\sqrt{1+x^2}\right)$ (b) $y=\frac{10}{4x^3-9x^2+6x}$

Ans: (a) increases on $(-\infty,\infty)$ (b) increases on $\left(\frac{1}{2},1\right)$ and decreases on $(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup(1,\infty)$

28) Find the equation of normal to the curve $y=(1+x)^y+\sin^{-1}(\sin^2 x)$ at $x=0$ Ans: x+y=1

29) If P_1 and P_2 are the lengths of the perpendiculars from origin on the tangent and normal to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ respectively Prove that $4P_1^2+P_2^2=a^2$

30) What angle is formed by the y-axis and the tangent to the parabola $y = x^2+4x-17$ at the point $P\left(5/2,-3/4\right)$? Ans: $\theta=\Pi/2-\tan^{-1}9$

31) A cone is circumscribed about a sphere of radius r. Show that the volume of the cone is maximum when its semi vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$

32) Find the interval in which the function f(x) is increasing or decreasing

$f(x)=x^3-12x^2+36x+17$ Ans: Increasing in x<2 or x>6
Decreasing 2<x<6

33) Find the equation of tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point (x_1,y_1) and show that the sum is intercepts on the axes is constant.

34) Find the point on the curve $y = x^3-11x+5$ at which the tangent has equation $y = x-11$
Ans: (2,—9)

35) Find the equation of all lines having slope —1 and that are tangents to the curve

$y=\frac{1}{x-1}, x\neq 1$ Ans: x+y+1=0 ; x+y—3=0

36) Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

37) Find the points on the curve $y = x^3 - 3x^2+2x$ at which the tangent to the curve is parallel to the line y — 2x + 3 = 0. Ans: (0,0) (2,0)

38) Show that the semi vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

39) Show that the semi vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$

40) Show that the right circular cone of least curved surface area and given volume is $\sqrt{2}$ times the radius of the base.

41) Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$

42) Find the absolute maximum and absolute minimum value of $f(x) = 2\cos x + x \quad x \varepsilon [0, \Pi]$

Ans: max at $x = \Pi/6$ and min at $x = 5\Pi/6$

43) Show that the volume of greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\Pi h^3 \tan^2 \alpha$

44) A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is 'P' cm. Show that the window will allow the maximum possible light only when the radius of the semi circle is $\frac{P}{\Pi + 4}$ cm

45) Find the area of the greatest isoscles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.

46) A rectangular window is surmounted by an equilateral triangle. Given that the perimeter is 16 cm. Find the width of the window so that the maximum amount of light may enter.

47) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle 30° is $\frac{4}{81}\Pi h^3$

48) Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $h/3$.

49) Of all the rectangles each of which has perimeter 40 metres find one which has maximum area. Find the area also.

50) Show that the rectangle of maximum area that can be inscribed in a circle of radius 'r' cms is a square of side $\sqrt{2}\, r$

51) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

52) Show that the semi vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

53) Given the sum of the perimeters of a square and a circle show that the sum of their areas is least when the side of the square is equal to the diameter of a circle.

54) Find the maximum slope of the curve $f(x) = 2x + 3x^2 - x^3 - 27$

55) Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

56) A point on the hypotenuse of a right angled triangle is at distances a and b from the sides. Show that the length of the hypotenuse is at least $\left(a^{2/3} + b^{2/3}\right)^{3/2}$

57) Sand is pouring from a pipe at the rate of $12 cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone in increasing when the height is 4 cm?

Ans : $\frac{1}{48\Pi}$ cm/sec.

58) A man 160 cm tall walks away from a source of light situated at the top of the pole 6m high at the rate of 1.1 m/sec. How fast is the length of the shadow increasing when he is 1m away from the pole. Ans: 0.4 cm/sec.

59) An edge of a variable cube is increasing at the rate of 5 cm/sec. How fast is the volume of the cube is increasing when edge is 10cm long? Ans: 1500 cm^3/sec.

60) A balloon which always remains spherical is being inflated by pumping in gas at the rate of 900 cm^3/sec. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15 cm. Ans: $\frac{1}{\Pi}$ cm/sec.

61) The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when the radius is 5 cm. Ans: 10 cm^2/sec.

62) The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.

Ans: $80\Pi cm^2/sec.$

63) Gas is escaping from a spherical balloon at the rate of 900 cm^3/sec. How fast is the surface area, radius of the balloon shrinking when the radius of the balloon is 30cm?

Ans: $\frac{dA}{dt} = 60cm^2/sec.$ $\frac{dr}{dt} = \frac{1}{4\Pi}$ cm/sec.

64) Water is passed into an inverted cone of base radius 5 cm and depth 10 cm at the rate of $\frac{3}{2}$ c.c./sec. Find the rate at which the level of water is rising when depth is 4 cm.

Ans: $\frac{3}{8\Pi}$ cm/sec.

65) Show that the function $f(x) = e^{2x}$ is strictly increasing on R.

66) Show that $f(x) = 3x^5 + 40x^3 + 240x$ is always increasing on R.

67) Find the interval in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is increasing or decreasing.

68) Find whether $f(x) = \cos\left(2x + \Pi/4\right) \frac{3\Pi}{8} < x < \frac{5\Pi}{8}$ is increasing or decreasing.

69) Find the interval in which the function $\frac{4.\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing or decreasing.

Ans: $\left(0, \Pi/2\right) \cup \left(\frac{3\Pi}{2}, 2\Pi\right)$

70) FInd the interval in which $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing.

Ans: Increasing : $2 < x < 3$ decreasing : $x > 3$ or $x < -2$

71) Find the interval on which the function $\frac{x}{\log x}$ is increasing or decreasing.

Ans: Increasing in $((e, \infty)$

decreasing for (0,1)U(1,e)

72) Find the points on the curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are parallel to x-axis and parallel to y-axis. Ans: $(0,\pm 5), (\pm 2,0)$

73) Find equation of tangents to the curve $x = a\cos^3\theta,\ y = b\sin^3\theta$ at $\theta = \Pi/4$

74) Find the equations of the normal lines to the curve $y = 4x^3-3x+5$ which are parallel to the line $9y+x+3 = 0$. Ans: $x+9y-55=0,\ x+9y-35=0$

75) FInd the equation of tangent and normal to the curve $y^2 = \frac{x^3}{4-x}$ at $(2, -2)$

Ans: $2x+y-2=0,\ x-2y-6=0$

76) FInd the equation of the tangent to the curve $\sqrt{x}+\sqrt{y} = a$ at the point $\left(\frac{a^2}{4},\frac{a^2}{4}\right)$

Ans: $2x+2y=a^2$

77) Find the angle between the parabolas $y^2=4ax$ and $x^2=4by$ at their point of intersection other than the origin. Ans: $\theta = \tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2\left(a^{2/3}+b^{2/3}\right)}\right]$

78) Using differentials find the appropriate value of $(82)^{1/4}$ Ans: 3.0092

79) If $y = x^4-10$ and if x changes from 2 to 1.97, what is the appropriate change in y ?

Ans: -0.96, y changes from 6 to 5.04

80) Find the appropriate change in volume of a cube when side increases by 1%. Ans: 3%

81) Use differentials to evaluate $\left(\frac{17}{81}\right)^{1/4}$ approximately. Ans: 0.677

82) Using differentials evaluate tan 44^o approximately, $1^o = 0.07145^o$. Ans: 0.9651

83) Find the approximate value of x if $2x^4-160 = 0$ Ans: 2.991

84) Find the maximum and minimum values of 'f', if any of the function $f(x) = |x|,\ x \in R$

85) Find the maximum and minimum value of $f(x) = |(\sin 4x+5)|$ without using derivatives.

86) The curve $y = ax^3+6x^2+bx+5$ touches the x-axis at P(-2,0) and cuts the y-axis at a point Q where its gradient is 3. Find a,b,c.

Ans: $a = -\frac{1}{2}\ b = \frac{-3}{4},\ c = 3$

87) FInd the local maxima and local minima if any of the function $f(x) = e^{5x}$

88) Find the maxima or minima if any of the function $f(x) = \frac{1}{x^2+2}$

Ans: local max at x = 0, value $\frac{1}{2}$

89) Without using derivatives find the maximum or minimum value of $f(x) = -|x+5| + 3$

Ans: max value 3, no minimum value

90) Without using derivatives find the maximum and minimum value of $f(x) = \sin 2x + 7$

Ans: Max. value 8, min. value 6

91) FInd whether $f(x) = e^x$ has maxima or minima. Ans: No maxima nor minima

92) At what point in the interval $[0,2\Pi]$ does the function Sin2x attain its maximum value?

93) Find the intervals in which f(x) = logcosx is strictly decreasing and strictly increasing.

Ans: decreasing $\left(0, \Pi/2\right)$, increasing $\left(\Pi/2, \Pi\right)$

94) Find the interval in which $y = x^2 e^{-x}$ is increasing. Ans: (0,2)

95) Find two positive numbers x and y such that their sum is 16 and sum of whose cubes is minimum.

Ans: 8,8

96) Find the local maximum and local minimum value of the function.

$f(x) = \sin x + \frac{1}{2}\cos 2x \text{ in } \left[0, \Pi/2\right]$ Ans: local max. value $\frac{3}{4}$ at $x = \frac{\Pi}{6}$

97) Two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base?

98) A poster is to contain $50cm^2$ of matter with borders of 4 cm at top and bottom and of 2 cm on each side. Find the dimensions if the total area of the poster is minimum.

99) Find the sides of a rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 16$

Ans: $4\sqrt{2}, 2\sqrt{2}$

100) Find the maximum profit that a company can make if the profit function is given by

$P(x) = 41 - 24x - 18x^2$ Ans: 49

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INTEGRATION

Objective Questions - choose the correct alternative

1) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx =$

(a) $\sin\sqrt{x}$ (b) $2\cos\sqrt{x}$ (c) $2\sin\sqrt{x}$ (d) $\frac{\sqrt{\cos x}}{x}$

2) $\int \frac{\tan(\log x)}{x}dx =$

(a) log cos(logx) (b) log sec(logx) (c) log sin(logx) (d) —logcos(logx)

3) $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx =$

(a) $2\sqrt{\tan x}$ (b) $2\sqrt{\cot x}$ (c) $\sqrt{\cot x}$ (d) $\sqrt{\tan x}$

4) $\int \frac{dx}{\sin x + \sqrt{3}\cos x} =$

(a) $\frac{1}{2}\log\tan\left[\frac{x}{2}+\frac{\Pi}{6}\right]$ (b) $\frac{1}{2}\log\left\{\operatorname{cosec}\left(x+\frac{\Pi}{3}\right)-\cot\left(x+\frac{\Pi}{3}\right)\right\}$

(c) $\frac{1}{2}\log\left\{\sec\left(x-\frac{\Pi}{6}\right)+\tan\left(x-\frac{\Pi}{6}\right)\right\}$ (d) $-\frac{1}{2}\log\left\{\operatorname{cosec}\left(x+\frac{\Pi}{3}\right)+\cot\left(x+\frac{\Pi}{3}\right)\right\}$

5) $\int \frac{\sin 2x dx}{a^2\cos^2 x + b^2\sin^2 x} =$

(a) $(b-a)\log(a^2\cos^2 x + b^2\sin^2 x)$ (b) $\frac{1}{b-a}\log(a^2\cos^2 x + b^2\sin^2 x)$

(c) $\frac{1}{b^2-a^2}\log(a^2\cos^2 x + b^2\sin^2 x)$ (d) $\frac{1}{a^2+b^2}\log(a^2\cos^2 x + b^2\sin^2 x)$

6) $\int \frac{e^x dx}{e^{2x}+1} =$

(a) $\log(e^x + e^{-x})$ (b) $\log(e^{2x}+1)$ (c) $\tan^{-1}(e^x)$ (d) $\tan^{-1}(2e^x)$

7) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx =$

(a) $\frac{2}{3}\left(\frac{x}{a}\right)^{3/2}$ (b) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}$ (c) $\frac{2}{3}\cos^{-1}\left(\frac{x}{a}\right)^{3/2}$ (d) $\frac{2}{3}\sin^{-1}\left(\frac{a}{x}\right)^{3/2}$

8) $\int x^3 e^{x^2} dx =$

(a) $x^2(e^{x^2}-1)$ (b) $\frac{1}{2}x^2(e^{x^2}-1)$ (c) $\frac{1}{2}e^{x^2}(x^2-1)$ (d) $\frac{1}{2}(e^{x^2}-1)$

9) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx =$

(a) $\sqrt{1-x^2}\sin^{-1}x$ (b) $x\sin^{-1}x$ (c) $x - \sqrt{1-x^2}\sin^{-1}x$ (d) $(\sin^{-1}x)^2$

10) $\int x\tan^{-1}x\,dx =$

(a) $\left(\frac{x^2+1}{2}\right)\tan^{-1}x - \frac{x}{2}$ (b) $\left(\frac{x^2+1}{2}\right) + \tan^{-1}x - x$ (c) $(x^2+1)\tan^{-1}x - x$ (d) $(x^2+1)\tan^{-1}x + x$

11) $\int \frac{x+\sin x}{1+\cos x}dx =$

(a) $\tan\frac{x}{2}$ (b) $x\tan\frac{x}{2}$ (C) $\cot\frac{x}{2}$ (d) $x\cot\frac{x}{2}$

12) $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x.\cos^2 x}dx =$

(a) $\tan x + \cot x$ (b) $\tan x - \cot x$ (c) $\tan x + \sec x$ (d) $\tan x + \mathrm{cosec}x$

13) $\int \frac{dx}{x\sqrt{1-x^3}} =$

(a) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1}\right|$ (b) $\frac{1}{3}\log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^3}+1}\right|$

(c) $\frac{2}{3}\log\left|\frac{1}{\sqrt{1-x^3}}\right|$ (d) $\frac{1}{3}\log\left|1-x^3\right|$

14) $\int \frac{dx}{x-X^3} =$

(a) $\frac{1}{2}\log\frac{x^2}{1-x^2}$ (b) $\frac{1}{2}\log\frac{1-x^2}{x^2}$ (c) $\log\frac{(1-x)}{x(1+x)}$ (d) $\log x(1-x^2)$

15) $\int \sqrt{1+x^2}\,d(x^2) =$

(a) $\frac{2}{3x}(1+x^2)^{3/2}$ (b) $\frac{2}{3}(1+x^2)^{3/2}$ (c) $\frac{2x}{3}(1+x^2)^{3/2}$ (d) $\frac{2x}{3}(1+x^2)$

16) $\int \frac{2xdx}{1+x^4} =$

(a) $\tan^{-1}(x^2)$ (b) $\frac{1}{2}\tan^{-1}x^2$ (c) $\log(1+x^4)$ (d) $\tan^{-1}\left(\frac{1}{x^2}\right)$

17) $\dfrac{3^x dx}{\sqrt{1-9^x}} =$

(a) $\log_e 3 \sin^{-1}(3^x)$ (b) $\dfrac{1}{\log_e 3}\sin^{-1}(3^x)$ (c) $\log_e 3 \sin^{-1}(3^{x/2})$ (d) $\dfrac{1}{\log_e 3}\sin^{-1}(3^{x/2})$

18) $\dfrac{1}{x(x^4-1)}dx =$

(a) $\log\left[\dfrac{x^4}{x^4-1}\right]$ (b) $\dfrac{1}{2}\log\left[\dfrac{x^2+1}{x^2+1}\right]$ (c) $\dfrac{1}{4}\log\left[\dfrac{x^4-1}{x^4}\right]$ (d) $\log\dfrac{x(x^2-1)}{x^2+1}$

19) $\displaystyle\int \frac{e^{\log\left(1+\frac{1}{x^2}\right)}dx}{x^2+\frac{1}{x^2}} =$

(a) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(x+\dfrac{1}{x}\right)$ (b) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x-\frac{1}{x}}{\sqrt{2}}\right)$ (c) $\sqrt{2}\tan^{-1}\left(x+\dfrac{1}{x}\right)$ (d) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{x}-x\right)$

20) $\displaystyle\int \frac{dx}{(2+x)\sqrt{1+x}} =$

(a) $2\tan^{-1}\sqrt{1+x}$ (b) $\dfrac{1}{2}\tan^{-1}\sqrt{1+x}$ (c) $\tan^{-1}\sqrt{1+x}$ (d) $\log\{(2+x)\sqrt{1+x}\}$

21) $\displaystyle\int \frac{dx}{x+\sqrt{x}} =$ (a) $\log(1+\sqrt{x})$ (b) $\dfrac{1}{2}\log(1+\sqrt{x})$ (c) $2\log(1+\sqrt{x})$ (d) $\log(x+\sqrt{x})$

22) Ssin2x log cosx dx =

(a) $\dfrac{1}{2}\cos^2 x - \cos^2 x \log\cos x$ (b) $\dfrac{1}{2}\cos^2 x + \cos^2 \log\cos x$ (c) $\cos^2 x.\log\cos x$ (d) $\cos^2 x(1-\log\cos x)$

23) If $\displaystyle\int \frac{dx}{(x+1)(x-2)} = A\log(x+1) + B\log(x-2) + C$, then

(a) A+B = 0 (b) AB = − (c) AB = $\dfrac{1}{9}$ (d) AB = −9

24) If $\displaystyle\int \tan^4 x\, dx = a\tan^3 x + b\tan x + cx$, then

(a) $a = \dfrac{1}{3}$ (b) b = −1 (c) a = 1 (d) c = 1

25) If $\displaystyle\int \frac{\cos x}{\cos(x-a)}dx = Ax + B\log\cos(x-\alpha)$, then

(a) $A = \cos\alpha$ (b) $B = \sin\alpha$ (c) $A = \sin\alpha$ (d) $B = \cos\alpha$

Answers :

1. (c) 2. (b), (d) 3. (a) 4. (a), (b), (c), (d) 5. (c) 6. (c) 7. (b) 8. (c) 9. (c) 10. (a) 11. (b) 12. (a) 13. (b) 14. (a) 15. (b) 16. (a) 17. (b) 18. (c) 19. (b) 20. (a) 21. (c) 22. (a) 23. (a), (c) 24. (a), (b), (d) 25. (a), (b).

EVALUATE

26) $\int |x|\,dx$ Ans: $\frac{x}{2}|x|+c$

27) $\int\left(10^x+10x+\frac{10}{x}+\frac{x}{10}+x^{10}+10^{10}\right)dx$

Ans: $\frac{10^x}{\log 10}+\frac{101}{20}x^2+10\log|x|+\frac{x^{11}}{11}+10^{10}x+c$

28) $\int\frac{(a^x+b^x)^2}{a^xb^x}dx$ Ans: $\frac{\left(\frac{a}{b}\right)^x}{\log a/b}+\frac{\left(\frac{b}{a}\right)^x}{\log b/a}+2x+c$

29) $\int\frac{1}{1+\sin x}dx$ Ans: $\tan x - \sec x+c$

30) $\int\cos^{-1}(\sin x)dx$ Ans: $\frac{\Pi}{2}x-\frac{x^2}{2}+c$

31) $\int 3^{-2x}e^{-2x}dx$ Ans: $\frac{(3e)^{-2x}}{-2\log 3e}$

32) $\int\frac{1}{\sin^2 x\cos^2 x}dx$ Ans: $\tan x - \cot x+c$

33) $\int\frac{e^{2\log x}-1}{e^{2\log x}+1}dx$ Ans: $x - 2\tan^{-1}x+c$

34) $\int \mathrm{Sin}^2\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx$ $-|\le x\le 1$ Ans: $x-\frac{x^3}{3}+c$

35) $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx$ Ans: $x+c$

36) $\int\frac{\sin 2x}{a^2\cos^2 x+b^2\sin^2 x}dx$ Ans: $\frac{1}{b^2-a^2}\log(a^2\cos^2 x+b^2\sin^2 x)+c$

37) $\int\frac{1+\sin 2x}{x+\sin^2 x}dx$ Ans: $\log|x+\sin^2 x|+c$

38) $\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$ Ans: $2\sqrt{\tan x}+c$

39) $\int\sqrt{\frac{a-x}{a+x}}dx$ Ans: $a\sin^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$

40) $\int\frac{1}{e^x-1}dx$ Ans: $\log|e^x-1|-x+c$

41) $\int\frac{1+\tan x}{x+\log\sec x}dx$ Ans: $\log|x+\log\sec x|+c$

42) $\int \frac{1}{1+\sqrt{x}}\,dx$ Ans : $2\log(\sqrt{x}+1)+c$

43) $\int x^2\sqrt[3]{2x-1}\,dx$ Ans : $\frac{3}{1120}(2x-1)^{4/3}(56x^2+24x+9)+c$

44) $\frac{dx}{\sqrt{\sin^3 x\,\sin(x+\alpha)}}$ Hint : Put $\sqrt{\frac{\sin(x+\alpha)}{\sin x}}=t$ Ans: $\frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+c$

45) $\int e^{\sin^2 x}\sin 2x\,dx$ Ans : $e^{\sin 2x}+c$

46) $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ Ans : $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$

47) $\int \frac{dx}{(x+1)\sqrt{x^2+1}}$, $x>-1$ Hint : $-x+1=\frac{1}{t}$ Ans : $-\frac{1}{\sqrt{2}}\log\left\{\frac{1-x\pm\sqrt{2+2x^2}}{x+1}\right\}+c^{l}$

48) $\int \frac{1}{4+5\cos x}dx$ Ans: $\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right|+c$

49) $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$ Ans : $\frac{18}{25}x+\frac{1}{25}\log|3\sin x+4\cos x|+c$

50) $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$ Ans : $\frac{2}{\Pi}\left\{\sqrt{x-x^2}-(1-2x)\sin^{-1}\sqrt{x}\right\}-x+c$

51) $\int \frac{1}{x^4+1}dx$ Ans: $\frac{1}{2\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

52) S$\sqrt{\tan\theta}\,d\theta$ Ans: $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\theta-1}{\sqrt{2}\tan\theta}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan\theta-\sqrt{2}\tan\theta+1}{\tan\theta+\sqrt{2}\tan\theta}\right|+c$

53) $\int 2^{2^{2^x}}2^{2^x}2^x\,dx$ Ans: $\frac{1}{(\log 2)^3}\,2^{2^{2^x}}+c$

54) $\int \frac{\sin x+\cos x}{\sin^4 x+\cos^4 x}dx$ Ans: $\frac{1}{\sqrt{2}}\left\{\frac{1}{2\sqrt{2}+1}\log\left|\frac{\sqrt{\sqrt{2}+1}+t}{\sqrt{\sqrt{2}+1}-t}\right|+\frac{1}{2\sqrt{\sqrt{2}-1}}\tan^{-1}\frac{t}{\sqrt{\sqrt{2}-1}}\right\}+c$

Where t = sinx — cosx

55) $\int \frac{8}{(x+2)(x^2+4)}dx$ Ans: $\log(x+2)-\frac{1}{2}\log(x^2+4)+\tan^{-1}\frac{x}{2}+c$

56) $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx$ Ans: $\log\left(\frac{1+\sin x}{2+\sin x}\right)+c$

57) $\int \frac{\sqrt{\cos 2x}}{\sin x}dx$ Ans: $\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+c$ Where t = $\sqrt{1-\tan^2 x}$

58) $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$ Ans : $\frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + c$

Hint: divided n r and D r by $\cos^2 x$

59) $\int \sqrt{\frac{x}{a^3 - x^3}}\,dx$ Ans : $\frac{2}{3}\sin^{-1}\frac{x^{3/2}}{a^{3/2}} + c$

60) $\int \frac{dx}{x(x^7+1)}$ Ans : $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right| + c$

61) $\int_{\Pi/3}^{\Pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{7/2}}\,dx$ Ans : $\frac{7}{3}$

62) $\int_{0}^{\Pi/4} \log(1+\tan\theta)\,d\theta$ Ans : $\frac{\Pi}{8}\log 2$

63) $\int_{0}^{\Pi/2} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2}$ Ans : $\frac{\Pi(a^2+b^2)}{4a^3b^3}$ Hint: divide Nr and Dr by $\cos^4 x$

64) $\int_{0}^{\Pi/3} \frac{\cos x}{3+4\sin x}\,dx$ Ans : $\frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$

65) $\int_{0}^{1} \frac{x}{1+x^4}\,dx$ Ans : $\frac{\Pi}{8}$

66) $\int_{0}^{3/2} |x\cos \Pi x|\,dx$ Ans : $\frac{5\Pi - 2}{2\Pi^2}$

67) $\int_{0}^{\Pi/2} \frac{1}{2\cos x + 4\sin x}\,dx$ Ans : $\frac{1}{\sqrt{5}}\log\frac{3+\sqrt{5}}{2}$

68) $\int_{0}^{2} |x^2 + 2x - 3|\,dx$ Ans : 4

69) Evaluate $\int_{-\Pi/2}^{\Pi/2} \cos^4 x\,dx$ Ans : $\frac{3\Pi}{8}$

70) $\int_{0}^{\Pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ Ans : $\frac{\Pi^2}{4}$

71) $\int_{0}^{\Pi} \log(1+\cos x)\,dx$ ans : $-\Pi\log 2$

72) $\int_0^{\Pi} \frac{x\tan x}{\sec x \operatorname{cosec} x}$ Ans : $\Pi/2$

73) $\int_0^1 x(1-x)^n dx$ Ans : $\frac{1}{n+1} - \frac{1}{n+2}$

74) $\int_0^{\Pi/2} \log\tan x$ Ans : $I = 0$

75) $\int^{0} \frac{xdx}{1+\sin x+\cos x}$ Ans : $\Pi/2 \log 2$

76) $\int_{-\Pi/2}^{\Pi/2} \cos^4 x\, dx$ Ans : $\frac{3\Pi}{8}$

77) $\int_{-5}^{8} |x+3|dx$

78) $\int_0^{2\Pi} \frac{\sin 2\theta}{a-b\cos\theta}d\theta$ $a > b > 0$ Ans: $I = 0$

79) $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$ Ans : $\frac{2}{3}\left(3^{3/2}-1\right)$

80) $\int_0^{\Pi/4} \sin 2x \sin 3x\, dx$ Ans : $\frac{3}{5}$

81) $\int_0^{\Pi} \frac{x\, dx}{a^2\cos^2 x + b^2\sin^2 x}$ Ans : $\frac{\Pi^2}{2ab}$

82) $\int_{-\Pi/4}^{\Pi/4} |\sin x|\, dx$ Ans : $2 - \sqrt{2}$

83) $\int_{-\Pi/4}^{\Pi/4} |\sin x|\, dx$ Ans : $2 - \sqrt{2}$

84) $\int_0^{\Pi/2} \frac{\sin x - \cos x}{1+\sin x\cos x}dx$ Ans : 0

85) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$ Ans : $\Pi/4$

86) $\int_{1}^{4}\left[|x-1|+|x-2|+|x-3|\right]dx$ Ans : $\frac{19}{2}$

87) $\int_{-\Pi}^{\Pi} x^{20}\sin^{9}x\,d\,x$ Ans : 0

88) $\int_{3}^{9}\frac{\sqrt{12-x}}{\sqrt{x}+\sqrt{12-x}}dx$ Ans : 3

89) $\int_{-1}^{1}\log\left(\frac{4-x}{4+x}\right)dx$ Ans : 0

90) $\int_{0}^{\Pi/2}\frac{1}{2\cos x+4\sin x}dx$ Ans : $\frac{1}{\sqrt{5}}\log\frac{3+\sqrt{5}}{2}$

91) $\int_{0}^{\Pi/2}\log(\cos\theta)d\theta$ Ans : $\frac{-\Pi}{2}\log 2$

92) $\int_{-\Pi}^{\Pi}\frac{2x(1+\sin x)}{1+\cos^{2}x}$ Ans : Π^{2}

93) $\int_{0}^{1}x\sqrt{\frac{1-x^{2}}{1+x^{2}}}dx$ Ans : $\frac{\Pi}{4}-\frac{1}{2}$

94) $\int_{0}^{\Pi/2}\frac{1}{\cos\left(x-\Pi/3\right)\cos\left(x-\Pi/6\right)}$ Ans : $2\log\sqrt{3}+\log 2$

95) $\int_{-\Pi}^{\Pi}\left(\sin^{-93}x+x^{295}\right)$ Ans : 0

96) $\int_{0}^{16}\frac{x^{1/4}}{1+x^{1/2}}dx$ Ans : $\Pi/4$

97) $\int_{0}^{\Pi/2}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$ Ans : $\Pi/4$

98) $\int_{0}^{1}\sqrt{\frac{1-x}{1+x}}dx$ Ans : $\frac{\Pi}{2}-1$

99) $\int_{0}^{\Pi/4}2\tan^{3}x\,dx$ Ans : 1-log2

100) $\int_{\Pi/6}^{\Pi/3}\frac{dx}{1+\sqrt{\tan x}}$ Ans : $\Pi/12$

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APPLICATION OF INTEGRALS

1) Find the area bounded by the curve y = 2 cosx and the x-axis from x = 0 to x = $x = 2\Pi$

Ans: 8 sq.units.

2) Find the area bounded by the x-axis part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates x = 2 and x = 4 If the ordinate at x = a divides the area into two equal parts find 'a'

Ans: Note 2<a<4 a = $2\sqrt{2}$

3) Find the area included between the parabolas $y^2=4ax$ and $x^2=4ay$ Ans: $16\frac{a^3}{3}$ sq.units.

4) Find the area of the segment cut off from the parabola $y^2=2x$ by the line y=4x-1

Ans: $\frac{9}{32}$ sq.units

5) Show that the area enclosed by the circle $x^2+y^2 = 64a^2$ and the parabola $y^2= 12ax$ is $a^2\left(\frac{16}{\sqrt{3}} + \frac{64\Pi}{\sqrt{3}}\right)$

6) Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

Ans: $\frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}} + \text{Sin}^{-1}\frac{1}{\sqrt{5}}\right] - \frac{1}{2}$

7) Find the area of the region bounded by the curve C. y = tanx the tangent drawn to C at $x = \Pi/4$ and the x-axis.

Ans: $\frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$

8) Find the area of the region lying above x-axis and included between the curves $x^2+y^2 = 2ax$ and $y^2 = ax$

Ans: $a^2\left(\frac{\Pi}{4} - \frac{2}{3}\right)$

9) Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1 + x^2}$ and find its area.

Ans: $\Pi - \frac{2}{3}$

10) Find the area of the smaller region bounded by the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $\frac{x}{4} + \frac{y}{3} = 1$

Ans: $\frac{\Pi}{3}$ sq.units.

11) Using integration find the area of the triangle ABC where A is (2,3) B(4,7) and C(6,2)

Ans: 4 sq.units.

12) Using integration find the area of the triangle ABC whose vertices are A(3,0), B(4,6) and C(6,2)

Ans: 8 sq.units.

13) Find the area included between the curves $(x-1)^2+y^2 = 1$ and $x^2+y^2 = 1$

Ans: $\left(\frac{2\Pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq.units.

14) Sketch the region common to the circle $x^2+y^2 = 25$ and the parabola $y^2 = 8x$ also find the area of the region using integration.

$$\text{Ans : } \left\{\frac{\sqrt{2}}{3}\left(\sqrt{41}-4\right)^{3/2} + \frac{25}{4}\Pi - \frac{25}{2}\sin^{-1}\left(\frac{\sqrt{41}-4}{5}\right)\right\}$$

15) Find the area of the circle $x^2 + y^2 = a^2$ Ans: Πa^2 sq.units.

16) Sketch the region of the ellipse and find its area using integration. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad a > b$

Ans: Πab sq.units.

17) Find the area of the region given by : $\{(x,y): x^2 \le y \le |x|\}$ Ans: $\frac{1}{3}$ sq.units

18) Find the area of the region

$\{(x,y): y^2 \le 4x, 4x^2+4y^2 = 9\}$ Ans: $\left\{\frac{\sqrt{2}}{6}+\frac{9\Pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right\}$ sq.units.

19) Find the area of the region bounded by the circle $x^2+y^2 = 16$ and the line $y = x$ in the first quadrant. Ans: 2Π sq.units.

20) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$ Ans: $\frac{ab}{4}(\Pi-2)$ sq.units.

21) Find the area bounded by the curve $y = \sin x$, x=axis and between $x = 0$, $x = \Pi$

Ans: 2 sq.units.

22) Sketch the graph of $y = |x-1|$ and evaluate $\int_{-2}^{4}|x-1|dx$ Ans: 9 sq.units.

23) Find the area of the region enclosed between the circles $x^2+y^2 = 1$ and $\left(x-\frac{1}{2}\right)^2+y^2=1$

$$\text{Ans: } \left(\frac{-2\sqrt{3}+\sqrt{15}}{16}-2\sin^{-1}\frac{1}{4}+\Pi\right)\text{sq.units.}$$

24) Draw the rough sketch of $y = \sin 2x$ and determine the area enclosed by the lines $x = \Pi/4$ and $x = \frac{3\Pi}{4}$

Ans: 1 sq.units.

25) Compute the area bounded by the lines $x+2y = 2$, $y-x = 1$ and $2x+y = 7$.

Ans: 6 sq.units.

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HOTS : DIFFERENTIAL EQUATIONS

1) Write the order and degree of the differential equation. $\left[1+(y^{I})^2\right]^{3/2} = y^{II}$

(Ans: 2,2)

2) Form the differential equation of the family of curves represented by the equation

$(x+a)^2 - 2y^2 = a^2$ (Ans: $x^2 + 2y^2 - 4xyy^{I} = 0$)

3) Write the order and degree of the diff. equation $\left(\dfrac{d^2y}{dx^2}\right)^{2/3} = \left(y + \dfrac{dy}{dx}\right)^{1/2}$

(Ans: 2,4)

4) Form the differential equation of circles represented by

$(x-\alpha)^2 + (y-\beta)^2 = r^2$ by eliminating α and β Ans: $\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^3 = r^2\left(\dfrac{d^2y}{dx^2}\right)^2$

5) Show that $Ax^2+By^2 = 1$ is a solution of the diff. equation. $x\left[y\left(\dfrac{d^2y}{dx^2}\right)+\left(\dfrac{dy}{dx}\right)^2\right] = y\dfrac{dy}{dx}$

6) Solve $(x+y+1)\dfrac{dy}{dx} = 1$ [Ans : $x = ce^{y} - y - 2$]

7) Solve : $(x+y)^2\dfrac{dy}{dx} = a^2$ [Ans : $(x+y)$ - $a\tan^{-1}\left(\dfrac{x+y}{a}\right) = x + c$]

8) Solve : $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$ [Ans : $y = xe^{cx}$]

9) Solve : ydx - $(x+2y^2)dy = 0$ [Ans : $\dfrac{x}{y} = 2y + c$]

10) Solve : $\left[x\sqrt{x^2+y^2} - y^2\right]dx + xydy = 0$ [Ans : $\sqrt{x^2+y^2} + \log\dfrac{y}{x} = Cx$]

11) Solve : $\dfrac{dy}{dx} = \cos^3 x \sin^4 x + x\sqrt{2x+1}$ [Ans : $y = \dfrac{1}{5}\sin^5 x - \dfrac{1}{7}\sin^7 x + \dfrac{1}{10}(2x+1)^{5/2} - \dfrac{1}{6}(2x+1)^{3/2} + c$]

12) Solve : $(x^3y^3 + xy)\dfrac{dy}{dx} = 1$ [Ans : $e^{y^{3/2}}\left(-\dfrac{1}{x}\right) = y^2e^{y^{5/2}} - 2e^{y^{3/2}} + c$]

13) Solve : $\left(x^3 - 3xy^2\right)dx = \left(y^3 - 3x^2y\right)dx$ [Ans : $x^2 - y^2 = C(x^2+y^2)^2$]

14) Solve : $\sqrt{1-y^2}\,dx = \left[\sin^{-1} y - x\right]dy$ [Ans : $x = \left(\sin^{-1}y - 1\right) + Ce^{\sin^{-1y}}$]

15) Solve the differential equation :

$(x^2y + y^2x)dy = (x^3 + y^3)dx$ [Ans : $-\dfrac{y}{x} = \log(x-y) + c$]

16) Solve : $\frac{dy}{dx}=\frac{y^2+y+1}{x^2+x+1}=0$ $[Ans:(x+y+1)=C(1\text{-}x\text{-}y\text{-}2xy)]$

17) Solve : $(x^3-3xy^2)dx=(y^3-3x^2y)dy$ $[Ans:x^2-y^2=c(x^2+y^2)^2]$

18) Solve : $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$, where $y(1)=1$ $[Ans:y^2=x^2+x-1]$

19) Solve : $\frac{xdx+ydy}{xdy-ydx}=\sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$ $\left[Ans:\sqrt{x^2+y^2}=a\sin\left(c+\tan^{-1}\frac{y}{x}\right)\right]$

(Hint : take $x=r\cos\theta, y=r\sin\theta$, so that $x^2+y^2=r^2$]

20) When the interest is compounded continuously, the amount of money invested increases at a rate proportional to its size. If Rs.1000 is invested at 10% compounded continuously, in how many years will the original investment double itself?

$[Ans:10\log_e^2$ years]

21) A population grows at the rate of 8% per year. How long does it take for the population to double?

$\left[Ans:\frac{25}{2}\log 2 \text{ years}\right]$

22) A wet porous substance in the open air losses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half of its moisture during the first hour, when will it have lost 95% moisture, weather conditions remaining the same.

$\left[Ans:\frac{\log 20}{\log 2}\right]$

23) The surface area of a balloon being inflated, changes at a rate proportional to time t. If initially its radius is 1 unit and after 3 seconds it is 2 units, find the radius after time 't'.

$\left[Ans:r=\sqrt{1+\frac{t^2}{3}}\right]$

24) A curve passing through the point (1,1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of p from the x-axis. Determine the equation of the curve.

$[Ans:x^2+y^2=2x]$

25) Newton's law of cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the (constant) temperature t of the surrounding medium. We can write it as

$\frac{dT}{dt}=-k(T-t),\ k>0$ constant.

A cup of coffee is served at 185°F in a room where the temperature is 65°F. Two minutes later the temperature of the coffee has dropped to 155°F. (log 3/4=0.144, log3 = 1.09872). Find the time required for coffee to have 105°F temperature.

(Ans: 7.63 min.]

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HOTS : VECTORS

1) Find the unit vector perpendicular to both the vectors

$\bar{a} = 4\bar{i} - \bar{j} - 3\bar{k}$ and $\bar{b} = 2\bar{i} + 2\bar{j} - \bar{k}$ Ans : $\frac{1}{3}\left(-\bar{i} + 2\bar{j} + 2\bar{k}\right)$

2) If $\bar{\alpha} = 3\bar{i} - \bar{j}$ and $\bar{\beta} = 2\bar{i} + \bar{j} - 3\bar{k}$. Express $\bar{\beta}$ as a sum of two vectors $\bar{\beta}_1$ & $\bar{\beta}_2$, where $\bar{\beta}_1$ is parallel to $\bar{\alpha}$ and $\bar{\beta}_2$ is perpendicular to $\bar{\alpha}$.

3) If $\bar{a} + \bar{b} + \bar{c} = 0$, show that $\bar{a} x \bar{b} = \bar{b} x \bar{c} = \bar{c} x \bar{a}$

4) Prove the triangle inequality $\left|\bar{a}x + \bar{b}\right| \leq \left|\bar{a}\right| + \left|\bar{b}\right|$

5) Prove Cauchy - Schawarz inequality : $\left(\bar{a}.\bar{b}\right)^2 \leq \left|\bar{a}\right|^2.\left|\bar{b}\right|^2$

6) If $\bar{a}$ and $\bar{b}$ are vectors, prove that $\left|\bar{a} x \bar{b}\right|^2 + \left(\bar{a}.\bar{b}\right)^2 = \left|\bar{a}\right|^2.\left|\bar{b}\right|^2$

7) Prove that angle in a semi-circle is a right angle.

8) If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that

a) $\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} + \hat{b}\right|$ b) $\tan\frac{\theta}{2} = \frac{\left|\hat{a} - \hat{b}\right|}{\left|\hat{a} + \hat{b}\right|}$

9) Show that the perpendicular of the point $\bar{c}$ from the line joining $\bar{a}$ & $\bar{b}$ is $\frac{\left|\bar{a} x \bar{b} + \bar{b} x \bar{c} + \bar{c} x \bar{a}\right|}{\left|\bar{b} - \bar{a}\right|}$

(Hint : use area of triangle = $\frac{1}{2}bh$)

10) Show that the area of the parallelogram having diagonals $3\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$ is $5\sqrt{3}$

11) Vectors $2\bar{i} - \bar{j} + 2\bar{k}$ and $\bar{i} + \bar{j} - 3\bar{k}$ act along two adjacent sides of a parallelogram. Find the angle between the diagonals of the parallelogram.

12) L and M are the mid-points of sides BC & DC of a paralellogram ABCD. Prove that

$\overline{AL} + \overline{AM} = \frac{3}{2}\overline{AC}$

13) Let $\bar{a}, \bar{b}$ & $\bar{c}$ be three vectors such that $\left|\bar{a}\right| = 3, \left|\bar{b}\right| = 4, \left|\bar{c}\right| = 5$ and each one of them being perpendicular to sum of the other two, find $\left|\bar{a} + \bar{b} + \bar{c}\right|$ [Ans : $5\sqrt{2}$]

14) Prove that the area of a paralellogram with diagonals $\bar{a}$ and $\bar{b}$ is $\frac{1}{2}\left|\bar{a} x \bar{b}\right|$

15) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are the p^{th}, q^{th} and r^{th} terms of an AP and

$\bar{u} = (q - r)\bar{i} + (r - p)\bar{j} + (p - q)\bar{k}$ & $\bar{v} = \frac{1}{a}\bar{i} + \frac{1}{b}\bar{j} + \frac{1}{c}\bar{k}$ then prove that $\bar{u}$ & $\bar{v}$ are orthogonal vectors.

16) In a triangle ABC, prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

17) Using vector method prove that :

a) sin(A+B) = sinAcosB + cosAsinB

b) sin(A-B) = sinAcosB — cosAsinB

c) cos(A+B) = cosAcosB — sinAsinB

d) cos(A-B) = cosAcosB + sinAsinB

18) Using vector method, show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

19) Prove that the altitudes of a triangle are concurrent.

20) Prove that the perpendicular bisectors of a triangle are concurrent.

21) Using vector method, prove that if the diagonals of a parallelogram are equal in length, then it is a rectangle.

22) Using vector method, prove that if two medians of a triangle are equal, then it is an isosceles.

23) Using vector method, show that the diagonals of a Rhombus bisect each other at right angles.

24) Prove by vector method, that the parallelogram on the same base and between the same parallels are equal in area.

25) If a, b & c are the lengths of the sides opposite respectively to the angles A, B & C of a ΔABC, using vector method show that

a) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ b) $a = b\cos C + C\cos B$

26) If D, E & F are the mid-points of the sides of a triangle ABC, prove by vector method that area of $\Delta DEF = \frac{1}{4}(\text{area of } \Delta ABC)$

27) If a, b & C are the lengths of the sides of a triangle, using vector method, show that its area is $\sqrt{s(s-a)(s-b)(s-c)}$

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HOTS : 3D GEOMETRY

1) Show that the line $\bar{r} = \left(2\bar{i} - 2\bar{j} + 3\bar{k}\right) + \lambda\left(\bar{i} - \bar{j} + 4\bar{k}\right)$ is parallel to the plane $\bar{r}.\left(\bar{i} + 5\bar{j} + \bar{k}\right) = 5$

2) Find the equation of the plane passing through the point (1,4,-2) and parallel to the plane $-2x+y-3z=7$ [Ans: $2x-y+3z+8=0$]

3) What are the conditions that the planes $a_1x+b_1y+c_1z = d_1$ & $a_2x+b_2y+c_2z = d_2$ are (i) parallel (ii) perpendicular to each other?

Ans: (i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (ii) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

4) Find the value of k for which the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = z$ intersect at a point? $\left[\text{Ans}: k = \frac{9}{2}\right]$

5) Find the equation of the plane passing through the point (1,6,3) and perpendicular to the plane $2x+3y-z = 7$ (Ans: $3x+y+9z = 36$)

6) Find the value of k for which the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are co-planar. [Ans: $k = 0, -3$}

7) Find the equation of the plane passing through the point (1,1,1) and perpendicular to each of the plane $x+2y+3z = 7$ and $2x-3y+4z = 0$ [Ans: $17x+2y-7z-12 = 0$]

8) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = 2$ intersect. Also find the point of intersection. [Ans: $(-1,-1,-1]$

9) Find the image of the point (1,2,3) in the plane $x+2y+4z = 38$. [Ans: (3,6,11)]

10) Find the equation of the plane passing through the points $(1,-1,2)$ and $(2,-2,2)$ and perpendicular to the plane $6x-2y+2z = 9$. [Ans: $x+y-2z+4 = 0$]

11) Find the foot of the perpendicular drawn from the point A(1,0,3) to the join of the points B(4,7,1) and C(3,5,3) $\left[\text{Ans}: \frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right]$

12) Find the length and co-ordinates of the foot of perpendicular from point (1,1,2) to the plane $2x-2y+4z+5 = 0$

$$\left[\text{Ans}: \frac{13\sqrt{6}}{12}, \left(-\frac{1}{12}, \frac{25}{12}, -\frac{1}{6}\right)\right]$$

13) Find the equation of the plane through the points $(-1,1,1)$ and $(1,-1,1)$ perpendicular to the plane $x+2y+2z = 5$ (Ans: $2x+2y-3z+3 = 0$)

14) Find the perpendicular distance of point (2,3,4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$$\left[\text{Ans}: \frac{1}{49}\sqrt{44541}\right]$$

15) The foot of the perpendicular drawn from the origin to the plane is (2,5,7). FInd the equation of the plane. [Ans: $2x+5y+7z = 78$]

16) Find the values of P so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. $\left[\text{Ans}:\frac{70}{11}\right]$

17) Find the shortest distance between two lines whose vector equations are

$\vec{r}=(1-t)i+(t-2)\bar{j}+(3-2t)\bar{k}$ and $\vec{r}=(s+1)\bar{i}+(2s-1)\bar{j}-(2s+1)\bar{k}$ $\left(\text{Ans}:\frac{8}{\sqrt{29}}\right)$

18) Find the vector equation of the plane through the intersection of the planes

$\bar{r}.(2\bar{j}+6\bar{j})+12=0$ & $\bar{r}.(3\bar{i}-\bar{j}+4\bar{k})=0$ which are at a unit distance from the origin.

$\left[\text{Ans}:\bar{r}.\left(-\bar{i}+2\bar{j}-2\bar{k}\right)+3=0\right]$

19) Find the equation of the line passing through the point (3,0,1) and parallel to the planes x+2y = 0 and 3y —z = 0 $\left[\text{Ans}:\bar{r}=(3\bar{i}+\bar{k})+\lambda(-2\bar{i}+\bar{j}+3\bar{k})\right]$

20) Find the reflection of the point (1,2,—1) in the plane 3x—5y+4z = 5 $\left[\text{Ans}:\left(\frac{73}{25},\frac{-6}{5},\frac{39}{25}\right)\right]$

21) Find the distance of the point (1,—2,3) from the plane x—y+z = 5 measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{2}=\frac{z+1}{-6}$ [Ans: 1]

22) Find the distance of the point (2,3,4) from the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ measured parallel to the plane 3x+2y+2z+5 = 0

23) A line makes angles α,β,γ and δ with the four diagonals of a cube. Prove that

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

24) A variable plane which remains at a constant distance 3p from the origin cuts the co-ordinate axes at A, B & C. Show that the locus of the centroid of the ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

25) A variable plane is at constant distance p from the origin and meet the axes in A, B & C. Show that the locus of the centroid of the tetrahedron ΔABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$

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LINEAR PROGRAMMING

4 Marks/6 marks

1) A toy company manufactures two types of doll; a basic version-doll A and a deluxe version doll B. Each doll of type B takes twice as long as to produce as one of type A, and the company would have time to make a maximum of 2000 per day if it produces only the basic version. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If company makes profit of Rs.3 and Rs.5 per doll, respectively, on doll A and B; how many each should be produced per day in order to maximize profit.

   Ans: Type A = 1000, Type B = 500, Max. profit = Rs.5500

2) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A, while each packet of the same quality of food Q contains 3 units of calcium, 20 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and almost 300 units of cholesterol. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A?

   Ans: 40 packets of food P and 15 packets of food Q Maximum at (40,15) = 285

3) An oil company has tow depots A and B with capacities of 7000L and 4000L respectively. The company is to supply oil to three petrol pumps D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

	Distance in km	
From/To	A	B
D	7	3
E	6	4
F	3	2

   Assuming that the transportation cost of 10 litres of oil is Rs.1 per km, how should the delivery be scheduled in order that the transportation cost is minimum.

   Ans: From A : 500, 3000 and 3500 litres, From B : 4000, 0, 0 litres to D, E and F respectively. Minimum cost = Rs.4400

4) A firm makes two types of furniture : chairs and tables. The contribution to profit for each product as calculated by the accounting department is Rs.20 per chair and Rs.30 per table. Both products are to be processed on three machines M_1, M_2 and M_3. The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Chair	Table	Available Time
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

   How should the manufacturer schedule his production in October to maximize profit.

   Ans: 3 chairs and 9 tables.

5) A farmer has a supply of chemical fertilizer of type I which contains 10% nitrogen and 6% phosphoric acid and type II fertilizer which contains 5% nitrogen and 10% phosphoric acid. After testing the soil conditions of a field, it is found that atleast 14 kg of nitrogen and 14 kg of phosphoric acid is required for a good crop. The fertilizer type I costs Rs.2.00 per kg and type II costs Rs.3.00 per kg. How many kilograms of each fertilizer should be used to meet the requirement and the cost be minimum.

   Ans: Minimum at (100,80) and is equal to Rs.440.

6) If a young man rides his motorcycle at 25 km/hr, he had to spend Rs.2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases at Rs.5 per km. He has Rs.100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as LPP and solve it graphically.

   Ans: Maximum at $\left(\frac{50}{3}, \frac{40}{3}\right)$ and is equal to 30 km.

7) Solve the following LPP graphically. Maximize or minimize Z = 3x+5y subject to

$$3x - 4y \geq -12$$
$$2x - y + 2 \geq 0$$
$$2x + 3y - 12 \geq 0$$
$$0 \leq x \leq 4$$
$$y \geq 2$$

Ans: Min. value 19 at (3,2) and Max. value 42 at (4,6)

8) Solve the following LPP graphically. Minimize Z = 3x+5y subject to

$$-2x + y \leq 4$$
$$x + y \geq 3$$
$$x - 2y \leq 2$$
$$x, y \geq 0$$

Ans : Minimum value is $\frac{29}{3}$ at $\left(\frac{8}{3}, \frac{1}{3}\right)$

9) Determine graphically the minimum value of the objective function.

$$Z = -50x + 20y$$

Subject to constraints

$$2x - y \geq -5$$
$$3x + y \geq 3$$
$$2x - 3y \leq 12$$
$$x \geq 0,\ y \geq 0$$

10) Find the maximum and minimum values of 5x+2y subject to constraints

$$-2x - 3y \leq -6$$
$$x - 2y \leq 2$$
$$6x + 4y \leq 24$$
$$-3x + 2y \leq 3$$
$$x \geq 0 \text{ and } y \geq 0$$

Ans : Max. value is 19 at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and

Min. value is 4.85 at $\left(\frac{3}{13}, \frac{24}{13}\right)$

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HOTS : HIGH ORDER THINKING QUESTIONS - MATHEMATICS
PROBABILITY

1) Urn A contain 1 white, 2 black, 3 red balls. Urn B contain 2 white, 1 black, 1 red balls. Urn C contains 4 white, 5 black, 3 red balls. Two balls are drawn from one of the Urn and found to be one white and one red. Find the probabilities that they come from Urns A, B or C.

16/39

2) A die is thrown 120 times and getting 1 or 5 is considered success. Find the mean, variance of number of successes. $\mu = 40, \sigma^2 = 26.7$

3) Given P(A) = 0.3, P(B) = 0.2, Find P(B/A) if A and B are mutually exclusive events. (0)

4) The parameters n and p are 12 and $1/3$ for a binomial distribution. Find standard deviation.

(1.63)

5) A man fires 4 bullets on a thief. The probability that the thief will be killed by one bullet is 0.6. Find the probability that the thief is still alive. $(0.4)^4$

6) In a hurdle race a player has to cross 10 hurdles. The probability that will clear each hurdle is $5/6$, what is the probability that he will knock down fewer than 2 hurdles. (0.4845

7) If on an average 1 ship in every 10 sinks, find the chance that out of 5 ships atleast 4 will arrive safely. (0.9185)

8) 4 persons are chosen at random from a group of 3 men, 2 women, 3 children. Find the probability that out of 4 choice, exactly 2 are children. $3/7$

9) Suppose X has a binomial distribution B (6, $1/2$) show that X = 3 is the most likely outcome.

10) In a binomial distribution, the sum of mean and variance is 42. Product is 360. FInd the distribution.

$\left(\frac{2}{5}+\frac{3}{5}\right)^{50}$

11) Given that the two numbers appearing on two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'. $\frac{1}{15}$

12) $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}\ P(\text{not A or not B})=\frac{1}{4}$ state whether A and B are independent. (No)

13) Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that cards are a king, queen and a jack. $\frac{6}{2197}$

14) Find the probability of throwing almost 2 sixes in 6 throws of a single dice. $\left(\frac{35}{18}\right)\left(\frac{5}{6}\right)^4$

15) Find the probability that sum of the numbers showing on the two dice is 8, given that atleast one dice doesn't show five. $\left(\frac{3}{25}\right)$

16) The mean and variance of a binomial distribution are 4 and $4/3$. Find $P(X \geq 1)$ $\left(\frac{728}{729}\right)$

17) 6 boys and 6 girls sit in a row at random. Find the probability that 1) The six girls sit together 2) The boys and girls sit alternatively. $\left(\frac{1}{132}\right)\left(\frac{1}{462}\right)$

18) If A, B, C are events associated with random expt. Prove that

$PtP(A \cup B \cup C) = P(A) + P(B) + P(c) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

19) Shreya visits the cities A, B, C and D at random. What is the probability that he visits

1) A before B 2) A just before B. $\left(\frac{1}{2}, \frac{5}{24}\right)$

20) What are the odds in favour of getting a '3' in a throw of die? What are the odds against getting 3? $\left(\frac{1}{5}, \frac{5}{1}\right)$

21) A pack of 52 cards were distributed equally among 4 players. Find the chance that 4 kings are held by particular player. $\left(\frac{11}{4165}\right)$

22) A fair die is rolled. The probability that the first 1 occurs at the even number of trails is $\left(\frac{5}{11}\right)$

23) If a 4 digit number > 5000 are randomly formed from digits 0,1,3,5,7. Find probability of forming a number divisible by 5 when (1) digits are repeated 2) digits are not repeated.

$\left(\frac{2}{5}\right)\left(\frac{3}{8}\right)$

24) A letter is chosen at random from the word "ASSASSINATION". Find the probability that the letter is a vowel. $\left(\frac{6}{13}\right)$

25) A fair die is rolled. The probability that the first 1 occurs at even number of trails is $\left(\frac{5}{11}\right)$

26) If three distinct numbers are chosen at randomly from first 100 natural nos. then the probability that all of them are divisible by 2 or 3 is $\left(\frac{4}{1155}\right)$

27) A coin is tossed 7 times. FInd the probability distribution of getting 'r' heads.

$$\left(7c_r\left(\frac{1}{2}\right)^7, r = 0,1,2\ldots\ldots7\right)$$

28) A company produces 10% defective items. Find the probability of getting 2 defective items in a sample of 8 items is $\frac{28x9^6}{10^8}$

29) Obtain the probability distribution of number of sixes in two tosses of a dice. Also find mean/ variance. $\left(25/36, 10/36, 1/36, 1/3, 5/18\right)$

30) A,B,C tosses a coin in turns. The first one to throw a 'head' wins game. What are their respective chances of winning. $\left(4/7, 2/7, 1/7\right)$

31) A man is known to speak truth 3 times out of 4. He throws a dice and reports that it is a six. Find the probability that it is actually a six. $\left(3/4\right)$

32) Suppose that a fair dice are tossed and let X represents "The sum of points". FInd the mean/ variance of X. $\left(7, \sqrt{2.41}\right)$

33) Find the probability that sum of nos. appearing and showing on two dice is 8, given that atleast one of the dice doesn't show 5. $\left(1/9\right)$

34) A tells lie is 30% cases, and B in 35% cases find the probability that both state same fact.

35) Two cards are drawn without replacement from a pack. Find the probability distribution of number of face cards. $\left[\frac{105}{221},\frac{96}{221},\frac{20}{221}\right]$

P(X) = 0 1 2

36) A man takes a step forward with probability 0.4 and backwards with a probability 0.6. Find the probability that after 11 steps he is just one step away from the starting point.

$(462) \times (0.24)^5$

37) Find the probability distribution of the sum of the numbers obtained when two dice are thrown once. (All 11 prob. distributions to be shown)

38) Two cards are drawn from a pack. Find the probability that number of aces are drawn. (Write probability distribution table)

39) Find the mean and variance of number of sines in two tosses of a die. $\left(\frac{1}{3}\right),\left(\frac{5}{18}\right)$

40) 6 coins are tossed simultaneously. Find the probability of getting 1) no heads 2) 3 heads

$\left(\frac{1}{64}\right),\left(\frac{5}{16}\right)$

41) If $2p(A) = P(B) = 5/13$ and $P(A/B) = 2/5$ Find $P(A \cap B)$

42) If E and F are events such that $P(F) = 1/4$ $P(F) = 1/2$ and $P(E \cap P) = 1/8$ find $P(\bar{E} \cap \bar{F})$

43) P(A speaks truth is) = $4/5$. A coin is tossed. 'A' reports that a head appears. The probability that actually there was a head is.

44) Two cards are drawn from a pack and kept out. Then one card is drawn from remaining 50 cards. Find the prob. that it is an ace. $\left(\frac{1}{13}\right)$

45) Two dice are thrown. Find the probability that the number appeared have a sum 8 if it is known that second dice always exhibits 4. $\left(\frac{1}{6}\right)$

46) If the second die always shows an odd no. find the conditional probability of getting a sum as 7, if a pair of dice is to be known. $\left(\frac{1}{6}\right)$

47) $P(A) = 1/2, P(B) = 7/12$ and $P(\text{not } A \text{ or not } B) = 1/4$. State whether A and B independent or not. (NO)

48) A die is thrown again and again untill three sixes are obtained. Find the probability of obtaining third six in sixth throw of dice. $\left(\frac{625}{23328}\right)$

49) Six dice are thrown 729 times. How many times do you expect atleast three dice to show 5 or 6. (233)

50) A random variable X has probability distribution P(X) of the following form where K is some number.

$$P(X) = \begin{cases} k & \text{if} \quad x = 0 \\ 2k & \text{if} \quad x = 1 \\ 3k & \text{if} \quad x = 2 \\ 0 & \text{otherwise} \end{cases} \qquad \left(\frac{1}{6}\right)\left(\frac{1}{2}\right)$$

Find (1) K (2) P (X > 2)

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MATHEMATICS

SET I

Time: 3 hrs. **Max. Marks: 100**

General Instructions

1. **All questions are compulsory.**
2. **The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.**
3. **All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.**
4. **There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.**
5. **Use of calculators is not permitted. You may ask for logarithmic tables, if required.**

Section A

1. Give an example to show that the relation R in the set of natural numbers, defined by
R = {(x, y), x, y Î N, x £ y^2} is not transitive.

2. Write the principal value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$.

3. Find x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$.

4. For what value of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?

5. A square matrix A, of order 3, has | A | = 5, find |A . adj A |.

6. Evaluate $\int 5^x dx$.

7. Write the value of $\int_{\pi/2}^{\pi/2} \sin^5 x dx$.

8. Find the position vector of the midpoint of the line segment joining the points A $\left(5\hat{i}+3\hat{j}\right)$ and B $\left(3\hat{i}-\hat{j}\right)$.

9. If $\vec{a} = 2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = \left(6\hat{i}+\lambda\hat{j}+9\hat{k}\right)$ and $\vec{a} \parallel \vec{b}$, find the value of l.

10. Find the distance of the point (a, b, c) from x - axis.

Section B

11. Let N be the set of all natural numbers and R be the relation in N × N defined by (a, b) R (c, d) if ad = bc. Show that R is an equivalence relation.

12. Prove that $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

OR

Solve for x: $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$.

13. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1 + a^2 + b^2 + c^2).$$

14. For what values of a and b, the function f defined

as: $f(x) = \begin{cases} 3ax+b, & \text{if } x<1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x>1 \end{cases}$

is continuous at x = 1.

15. If $x^y + y^x = a^b$, find $\frac{dy}{dx}$.

OR

If x = a (cos t + t sint) and y = b (sin t – t cost), find $\frac{d^2y}{dx^2}$.

16. Find the intervals in which the following function is strictly increasing or strictly decreasing $f(x) = 20 - 9x + 6x^2 - x^3$.

OR

For the curve $y = 4x^3 - 2x^5$, find all points at which the tangent passes through origin.

17. Evaluate: $\int \dfrac{\sin x + \cos x}{\sqrt{\sin x . \cos x}}\, dx$.

OR

Evaluate: $\int e^x \left(\dfrac{x^2+1}{(x+1)^2}\right) dx$

18. Form the differential equation of the family of circles having radii 3.

19. Solve the following differential equation:

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0.$$

20. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

21. Find whether the lines $\vec{r} = \left(\hat{i}-\hat{j}-\hat{k}\right) + \lambda\left(2\hat{i}+\hat{j}\right)$ and $\vec{r} = \left(2\hat{i}-\hat{j}\right)+\mu\left(\hat{i}+\hat{j}-\hat{k}\right)$ intersect or not. If intersecting, find their point of intersection.

22. Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls. Find the probability distribution of number of green balls drawn.

Section C

23. If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the following system of equations:

$$2x + y + 3z = 3$$
$$4x - y \quad = 3$$
$$-7x + 2y + z = 2$$

OR

Using elementary transformations, find the inverse of the matrix: $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$

24. If the length of three sides of a trapezium, other than the base are equal to 10 cm each, then find the area of trapezium when it is maximum.

25. Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

26. Evaluate $\int_1^2 \left(x^2+x+2\right) dx$ as a limit of sums.

OR

Evaluate $\int_0^1 \sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right) dx,$

$$0 \le x \le 1$$

27. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane $2x - y + z + 3 = 0$. Find also, the image of the point in the plane.

28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by the executive class. Determine how many tickets of each type must be sold, in order to maximise profit for the airline. What is the maximum profit? Make an L. P. P. and solve it graphically.

29. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball drawn is red, what is the probability that bag B was picked up?

SET II

Time 3 hrs. **Max. marks: 100**

General Instructions same as set I.

Section A

1. Write the number of all one-one functions from the set A = {a, b, c} to itself.

2. Find x if $\tan^{-1} 4 + \cot^{-1} x = \dfrac{\pi}{2}$.

3. What is the value of $| 3\, I_3 |$, where I_3 is the identity matrix of order 3?

4. For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

5. If A is a matrix of order 2 × 3 and B is a matrix of order 3 × 5, what is the order of matrix (AB)¢or $(AB)^T$?

6. Write a value of $\int \frac{dx}{\sqrt{4-x^2}}$.

7. Find f(x) satisfying the following:
$\int e^x (\sec^2 x + \tan x)\, dx = e^x f(x) + C$.

8. In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA.

9. Find the value of l for which the vector $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\lambda\hat{j}-3\hat{k}$ are perpendicular to each other.

10. Find the value of l such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$.

Section B

11. Show that the function f: R ® R defined by $f(x) = 2x^3 - 7$, for x Î R is bijective.

OR

Let f, g: R ® R be defined as f (x) = | x | and g (x) = [x] where [x] denotes the greatest integer less than or equal to x. Find fog $\left(\frac{5}{2}\right)$ and gof $\left(-\sqrt{2}\right)$.

12. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 =$ p.

13. If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}.

14. Show that f(x) = | x – 3 |, ∀ x Î R, is continuous but not differentiable at x = 3.

OR

If $\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

15. Verify Rolle's Theorem for the function f, given $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

16. Using differentials, find the approximate value of $\sqrt{25.2}$.

OR

Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the triangle decreasing when the two sides are equal to 'a'.

17. Evaluate $\int_{-1}^{\frac{1}{2}} |x\cos(\pi x)|\, dx$.

18. Solve the following differential equation:

$$ye^{x/y}\, dx = \left(xe^{x/y} + y\right) dy$$

19. Solve the following differential equation:
$(1 + y + x^2y)\, dx + (x + x^3)\, dy = 0$, where y = 0 when x = 1

20. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2\left(\vec{b}\times\vec{c}\right)$.

21. Show that the four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar. Also, find the equation of the plane containing them.

22. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails.

OR

How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%.

Section C

23. Using properties of determinants, show that

$$D = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+b)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

24. The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

OR

A helicopter is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest distance between the soldier and the helicopter.

25. Evaluate: $\int \frac{1}{\sin x\,(5-4\cos x)}\,dx$

OR

Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

26. Using integration, find the area of the region

$$\left\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\right\}$$

27. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

28. From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?

29. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfying the requirements. Make it an LPP and solve it graphically.

SET III

Time: 3 hrs. **Max. Marks: 100**

General instructions same as set I.

Section A

1. Write the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

2. Write the range of the principal branch of $\sec^{-1}(x)$ defined on the domain R – (– 1, 1).

3. Find x if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$.

4. If A is a square matrix of order 3 such that | adj A | = 64. Find | A¢| .

5. If A is a square matrix satisfying $A^2 = 1$, then what is the inverse of A?

6. If f(x) = sin x°, find $\frac{dy}{dx}$

7. What is the degree of the following differential equation?

$$y\,\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = x\left(\frac{d^3y}{dx^3}\right)^2$$

8. If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a}\times\vec{b}| = 6$.

10. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axes.

Section B

11. Show that the relation R in the set A = {x: x Î Z, 0 £ x £ 12} given by R = {(a, b): |a–b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1.

12. Solve for x:

$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x),\ 0 < x < \frac{\pi}{2}.$$

OR

Show that $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= \frac{x+y}{1-xy}$, $|x| < 1, y > 0, xy < 1$.

13. If none of a, b and c is zero, using properties of determinants prove that:

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3.$$

14. If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

15. If $y = \left(x+\sqrt{x^2-1}\right)^m$, then show that

$$(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y = 0$$

16. Find all the points of discontinuity of the function $f(x) = [x^2]$ on $[1, 2)$, where [ ] denotes the greatest integer function.

OR

Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ w. r. t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

17. Evaluate: $\int\frac{1}{\cos(x-a)\cos(x-b)}dx$

OR

Evaluate: $\int x(\log x)^2 . dx$

18. Evaluate: $\int\frac{x}{x^3-1}dx$

19. Using properties of definite integrals, evaluate $\int_0^{\pi}\frac{x\,dx}{4-\cos^2 x}$.

20. The dot products of a vector with the vectors $\hat{i}-3\hat{k}$, $\hat{i}-2\hat{k}$ and $\hat{i}+\hat{j}+4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

21. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and (2, 3, 5). Also, find the perpendicular distance of the plane from the origin.

OR

Find the equation of the perpendicular drawn from the point P (2,4,–1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

22. A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If occurrence of an even number is considered a success, then write the probability distribution of number of successes. Also find the mean number of successes.

Section C

23. Using matrices, solve the following system of equations:

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z} = 4$; $\frac{2}{x}+\frac{1}{y}-\frac{3}{z} = 0$; $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 2$, $x \neq 0, y \neq 0, z \neq 0$.

24. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semivertical angle α, is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

OR

Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$ and $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

25. Find the area of the region: $\{(x, y): 0 \le y \le x^2, 0 \le y \le x+2; 0 \le x \le 3\}$.

26. Find the particular solution of the differential equation $(xdy-ydx)y.\sin\left(\frac{y}{x}\right) = (ydx+xdy)x\cos\frac{y}{x}$, given that $y = \pi$ when $x = 3$.

27. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = \left(-3\hat{i}+\hat{j}+5\hat{k}\right) + \lambda\left(3\hat{i}-\hat{j}+5\hat{k}\right)$. Also, show

that the plane contains the line

$$\vec{r} = \left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}-5\hat{k}\right).$$

28. A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit on each unit of A is Rs. 20 and on B is Rs. 15. How many units of A and B should be produced to maximise the profit. Form an L. P. P. and solve it graphically.

29. Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

OR

In an examination, 10 questions of true - false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true' and if it falls tails, he answers 'false'. Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

Answers

SET I

1. (8, 3) Î R, (3, 2) Î R but (8, 2) Ï R.

2. $\frac{\pi}{3}$ $\left(\because \cos\frac{5\pi}{3} = \cos\left(2\pi-\frac{\pi}{3}\right) = \cos\frac{\pi}{3}\right)$

3. $x = 4$

4. $a = \frac{4}{3}$ $\left(\because \begin{vmatrix} 2a & -1 \\ -8 & 3 \end{vmatrix} = 0 \Rightarrow 6a = 8, a = \frac{4}{3}\right)$

5. 125 $(\because |A||\text{adj } A| = |A|^n)$

6. $\frac{5^x}{\log 5} + C$

7. Zero (∵ sinx is an odd function.)

8. $4\hat{i}+\hat{j}$

9. $l = -3$

10. $\sqrt{b^2+c^2}$

11. For any (a, b) Î N × N, ab = ba
Þ (a, b) R (a, b). Thus R is reflexive.
Let (a, b) R (c, d) for any a, b, c, d Î N
\ ad = bc
Þ cb = da Þ (c, d) R (a, b).
\ R is symmetric.
Let (a, b) R (c, d) and (c, d) R (e, f)
for a, b, c, d, e, f Î N
then ad = bc and cf = de
Þ adcf = bcde or af = be Þ (a, b) R (e, f)
\ R is transitive.
Since, R is reflexive, symmetric and transitive, R is an equivalence relation.

12. $\text{L. H. S} = \tan^{-1}\left(\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\frac{17}{34}\right)$

$$= \tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\left(2\tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \frac{1}{2}\cos^{-1}\left[\dfrac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}\right] = \frac{1}{2}\cos^{-1}\left[\dfrac{1-\frac{1}{4}}{1+\frac{1}{4}}\right]$$

$$= \frac{1}{2}\cos^{-1}\frac{3}{5} = \text{RHS}.$$

OR

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right) = \cos(2\sin^{-1}x)$

$\Rightarrow (1-x) = \cos(2\alpha)$ where $\sin^{-1}x = \alpha$ or $x = \sin\alpha$

$\Rightarrow (1-x) = 1 - 2\sin^2\alpha = 1 - 2x^2$, $\therefore 2x^2 - x = 0$

$\Rightarrow x(2x-1) = 0$

$\Rightarrow x = 0, \frac{1}{2}$

Since $x = \frac{1}{2}$ does not satisfy the given equation $\therefore x = 0$.

13. LHS $= \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix}$

$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$

$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$

$= (1+a^2+b^2+c^2) \cdot 1 = (1+a^2+b^2+c^2)$

14. LHL $= \lim_{x\to 1^-} f(x) = 3a + b$,

RHL $= \lim_{x\to 1^+} f(x) = 5a - 2b$, $f(1) = 11$

$\therefore 3a + b = 11$ ------ (i)

$5a - 2b = 11$ ------ (ii)

Solving (i) and (ii) we get $a = 3, b = 2$

15. Put $x^y = u$ and $y^x = v$

$\therefore u + v = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$ ----- (1)

$\log u = y\log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$

$\therefore \frac{du}{dx} = y \cdot x^{y-1} + x^y \cdot \log x \cdot \frac{dy}{dx}$

$\log v = x\log y \Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$

$\therefore \frac{dv}{dx} = xy^{x-1} \cdot \frac{dy}{dx} + y^x \cdot \log y$

$(1) \to y \cdot x^{y-1} + x^y \log x \frac{dy}{dx} + xy^{x-1}\frac{dy}{dx} + y^x \log y = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y \cdot x^{y-1} + y^x \cdot \log y}{x^y \cdot \log x + xy^{x-1}}$

OR

$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$

$\frac{dy}{dt} = b(\cos t + t\sin t - \cos t) = bt\sin t$

$\frac{dy}{dx} = \frac{b}{a} \cdot \tan t \Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}\sec^2 t \cdot \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{b}{a}\sec^2 t \cdot \frac{1}{at\cos t} = \frac{b\sec^3 t}{a^2 t}$.

16. $f'(x) = 0 \Rightarrow -9 + 12x - 3x^2 = 0$

$\Rightarrow (-3)(x-1)(x-3) = 0$

$\Rightarrow x = 1, x = 3$

$\therefore$ The intervals are $(-\infty, 1), (1, 3), (3, \infty)$. (Since $f(x)$ to be strictly decreasing if $f'(x) < 0$ and strictly increasing if $f'(x) > 0$)

Getting $f(x)$ to be strictly decreasing in $(-\infty, 1) \cup (3, \infty)$ and strictly increasing in $(1, 3)$.

OR

Let (x_1, y_1) be a point on the given curve, the tangent at which passes through origin.

\ Slope of tangent = $\frac{y_1}{x_1}$ ----- (i)

Also, $\frac{dy}{dx}$ = $12x^2 - 10x^4$

Þ slope of tangent = $12x_1^2 - 10x_1^4$ ----- (ii)

Þ $\frac{y_1}{x_1} = 12x_1^2 - 10x_1^4$ or $y_1 = 12x_1^3 - 10x_1^5$

Þ $4x_1^3 - 2x_1^5 = 12x_1^5 = 12x_1^3 - 10x_1^5$

Solving to get $x_1 = 0$ or $1 - x_1^2 = 0$ ie., $x_1 = \pm 1$

Hence the required points are (0, 0), (1, 2) and (– 1, – 2).

17. Put (sin x – cos x) = t we get (cos x + sin x) dx = dt and $\sin^2 x + \cos^2 x - 2 \sin x \cos x = t^2$

or $\sin x \cos x = \frac{1}{2}(1 - t^2)$

\ Given integral = $\sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \cdot \sin^{-1} t + C$

$= \sqrt{2}\,\sin^{-1}(\sin x - \cos x) + C$

OR

$$\int e^x \cdot \frac{x^2+1}{(x+1)^2}\,dx = \int e^x \cdot \frac{\left[(x+1)^2 - 2x\right]}{(x+1)^2}\,dx$$

$$= \int e^x\,dx - 2\int e^x \cdot \frac{(x+1-1)}{(x+1)^2}\,dx$$

$$= e^x - 2\int\left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] e^x dx$$

$$= e^x - 2 \cdot \frac{e^x}{x+1} + C$$

[using $\int e^x\left(f(x) + f'(x)\right)dx = e^x f(x) + C$].

18. The equation of the family of circles is

$(x - a)^2 + (y - b)^2 = 9$ ----- (i)

Þ $2(x - a) + 2(y - b)\frac{dy}{dx} = 0$

or $(x - a) = -(y - b)\frac{dy}{dx}$ ------ (ii)

Þ $1 + (y - b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Þ $(y - b) = -\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$ --------- (iii)

From (ii), $(x - a) = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}} \cdot \frac{dy}{dx}$

Putting in (i) we get

$$\left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \cdot \left(\frac{dy}{dx}\right)^2 + \left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 = 9$$

or $\left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \cdot \left[\left(\frac{dy}{dx}\right)^2 + 1\right] = 9$

Þ $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = 9\left(\frac{d^2y}{dx^2}\right)^2$.

19. Given differential equation can be written as

$\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$

Þ $\frac{\sqrt{1+x^2}}{x}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$

\ $\int \frac{y}{\sqrt{1+y^2}}dy = -\int \frac{\sqrt{1+x^2}}{x^2}\,x dx$

Putting $1 + y^2 = u^2$ and $1 + x^2 = v^2$ we get ydy = udu and xdx = vdv

$$\therefore \int \frac{u\,du}{u} = -\int \frac{v.vdv}{v^2-1} = -\int \frac{v^2-1+1}{v^2-1}\,dv$$

$$= \int\left(1+\frac{1}{v^2-1}\right)dv$$

$$u = -v - \frac{1}{2}\log\left|\frac{v-1}{v+1}\right| + C$$

$$\text{or } \sqrt{1+y^2} = -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + C.$$

20. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vector such that $\vec{a}+\vec{b} = \vec{c}$

$$\therefore \left|\vec{a}+\vec{b}\right| = 1 \Rightarrow 1 = \left|\vec{a}+\vec{b}\right|^2 = \vec{a}^2+\vec{b}^2+2\vec{a}.\vec{b}$$

$$= 2+2\vec{a}.\vec{b}$$

$$\Rightarrow 2\left(\vec{a}.\vec{b}\right) = 1-2 = -1 \text{ ------- (i)}$$

$$\text{Now } \left|\vec{a}-\vec{b}\right|^2 = \vec{a}^2+\vec{b}^2-2\vec{a}.\vec{b} = 1+1-(-1) = 3$$

$$\Rightarrow \left|\vec{a}-\vec{b}\right| = \sqrt{3}$$

21. Given lines are $\vec{r} = (1+2\lambda)\vec{i}+(-1+\lambda)\vec{j}-\hat{k}$ and

$$\vec{r} = (2+\mu)\,\vec{i}+(-1+\mu)\,\vec{j}-\mu\vec{k}$$

If lines are intersecting, then for some value of λ and μ,

$1+2\lambda = 2+\mu$, ---- (i)

$-1+\lambda = -1+\mu$ ---- (ii)

$-1 = -\mu$ ------ (iii)

Solving (ii) and (iii) we get $\lambda = 1$, $\mu = 1$, which satisfy (i) hence the lines are intersecting and point of intersection is $(3, 0, -1)$.

22. Let X denotes the random variable, 'number of green balls',

X	0	1	2	3
P (X)	$\frac{{}^5C_3}{{}^9C_3} = \frac{5}{42}$	$\frac{{}^5C_2 . {}^4C_1}{{}^9C_3} = \frac{10}{21}$	$\frac{{}^5C_1 . {}^4C_1}{{}^9C_3} = \frac{5}{14}$	$\frac{{}^4C_3}{{}^9C_3} = \frac{1}{21}$.

23. $|A| = 2(-1) - 1(4) + 3(1) = -3 \neq 0$

$$A^{-1} = \frac{1}{|A|}\text{ adj } A.$$

The cofactors are

$A_{11} = -1$, $A_{12} = -4$, $A_{13} = 1$

$A_{21} = 5$, $A_{22} = 23$, $A_{23} = -11$

$A_{31} = 3$, $A_{32} = 12$, $A_{33} = -6$

$$\therefore A^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}$$

Given equations can be written as

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \text{ or } A . X = B$$

$$\therefore X = A^{-1} . B$$

$$\text{ie, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ -27 \\ 14 \end{pmatrix}$$

$\therefore x = -6, y = -27, z = 14$

OR

$$\text{Let } A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \text{ then}$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} A \quad R_3 \to R_3 - 3R_1$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} A \quad R_2 \leftrightarrow R_3$$

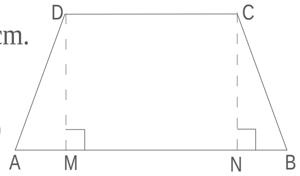
$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 1 \\ 6 & 1 & -2 \end{pmatrix} A \quad R_3 \to R_3 - 2R_2$$

$$\begin{pmatrix}1&0&0\\0&1&-2\\0&0&1\end{pmatrix}=\begin{pmatrix}-2&0&1\\-3&0&1\\6&1&-2\end{pmatrix}A \quad R_1 \to R_1+R_2$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{pmatrix}A \quad R_2 \to R_2+2R_3$$

Hence $A^{-1}=\begin{pmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{pmatrix}$.

24. AD = DC = BC = 10 cm.



$\Delta ADM \cong \Delta BCN$

$\therefore$ AM = BN = x (say)

$\therefore$ DM = $\sqrt{10^2-x^2}$

Area (A) = $\frac{1}{2}(10+10+2x)\sqrt{100-x^2}$

$= (10+x)\sqrt{100-x^2}$

Let $S = A^2 = (10+x)^2(100-x^2)$

$\frac{dS}{dx} = 0 \Rightarrow (10+x)^2(-2x)+(100-x^2)\,2\,(10+x)=0$

$\Rightarrow (10+x)^2(-2x+20-2x)=0 \Rightarrow x=5$

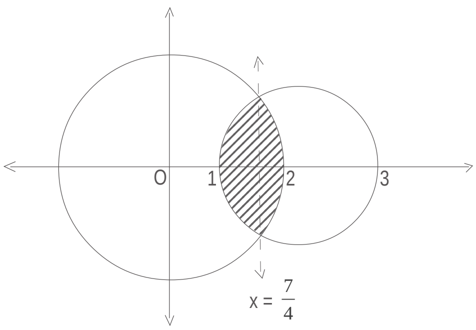
$\frac{d^2S}{dx^2} = (10+x)^2(-4)+(20-4x)\,2\,(10+x)<0$ at

$x = 5$

$\therefore$ For maximum area, x = 5

Maximum area = $15\sqrt{75} = 75\sqrt{3}$ cm^2.

25.



Solving $x^2+y^2=4$ and $(x-2)^2+y^2=1$

we get $x=\frac{7}{4}$

$\therefore$ Required area

$$=2\left[\int_{7/4}^{2}\sqrt{4-x^2}\,dx+\int_{1}^{7/4}\sqrt{1-(x-2)^2}\,dx\right]$$

$$=2\left[\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_{\frac{7}{4}}^{2}\right.$$

$$\left.+\left[\frac{x-2}{2}\sqrt{1-(x-2)^2}+\frac{1}{2}\sin^{-1}(x-2)\right]_{1}^{\frac{7}{4}}\right]$$

$$=2\left[\pi-\frac{7}{8}\frac{\sqrt{15}}{4}-2\sin^{-1}\frac{7}{8}\right.$$

$$\left.+\left(-\frac{1}{8}.\frac{\sqrt{15}}{4}+\frac{1}{2}\sin^{-1}\left(-\frac{1}{4}\right)+\frac{1}{2}.\frac{\pi}{2}\right)\right]$$

$$=\frac{5\pi}{2}-\frac{\sqrt{15}}{2}-\sin^{-1}\left(\frac{1}{4}\right)-4\sin^{-1}\left(\frac{7}{8}\right) \text{ sq. unit.}$$

26. Here $f(x) = (x^2+x+2)$, $h=\frac{b-a}{n}=\frac{1}{n}$

$$\int_1^2 f(x)\,dx = \lim_{n\to\infty}\frac{1}{n}.[f(1)+f(1+h)+f(1+2h)$$

$$+ \dots + f[1+(n-1)h]]$$

$$=\lim_{n\to\infty}\frac{1}{n}.[4+(4+3h+h^2)+(4+6h+4h^2)+\dots$$

$$+(4+(n-1)3h+(n-1)^2h^2]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{3}{n}.\frac{n(n-1)}{2}+\frac{1}{n}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{3}{2}\left(1-\frac{1}{n}\right)+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\right]$$

$$=4+\frac{3}{2}+\frac{1}{3}=\frac{24+9+2}{6}=\frac{35}{6}.$$

OR

Put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$

$\therefore\ \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

$= \sin^{-1}[\sin\alpha\cos\beta - \cos\alpha\sin\beta]$

$= \sin^{-1}[\sin(\alpha-\beta)] = \alpha - \beta = \sin^{-1}x - \sin^{-1}\sqrt{x}$

$$\therefore \text{ Given integral} = \int_0^1 \left(\sin^{-1}x - \sin^{-1}\sqrt{x}\right) dx$$

$$= \int_0^1 \sin^{-1}x\, dx - \int_0^1 \sin^{-1}\sqrt{x}\, dx$$

$$= \left[x.\sin^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1-x^2}} dx$$

$$- \left[x.\sin^{-1}\sqrt{x}\right]_0^1 + \int_0^1 \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}.x\, dx$$

$$= \frac{\pi}{2} + \left[\sqrt{1-x^2}\right]_0^1 - \frac{\pi}{2} + \frac{1}{2}\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$= -1 + \frac{1}{2}\int_1^0 \frac{-\sqrt{1-t^2}\,.2t\,dt}{t} \qquad [1 - x = t^2, dx = -2t\, dt]$$

$$= -1 + \int_0^1 \sqrt{1-t^2}\, dt = 1 + \left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right]_0^1$$

$$= \left(-1 + \frac{\pi}{4}\right).$$

27. Let Q be the foot of perpendicular from P to the plane and $P'(x, y, z)$ be the image of P in the plane.

$\therefore$ The equations of line through P and Q is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$$

The coordinates of Q (for some value of λ) are

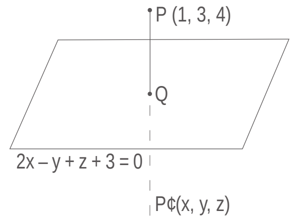
$(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Since Q lies on the plane

$\therefore\ 2(2\lambda + 1) - 1(-\lambda + 3) + (\lambda + 4) + 3 = 0$

Solving to get $\lambda = -1$

$\therefore$ Coordinates of foot of perpendicular (Q) are $(-1, 4, 3)$.



Perpendicular distance (PQ)

$$= \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6} \text{ units}$$

Since Q is the mid point of PP'

$$\therefore\ \frac{x+1}{2} = -1,\quad \frac{y+3}{2} = 4,\quad \frac{z+4}{2} = 3$$

$\Rightarrow\ x = -3, y = 5, z = 2$

$\therefore$ Image of P is $(-3, 5, 2)$.

28. Let number of executive class tickets to be sold, be x and that of economy class be y.

$\therefore$ LPP becomes:

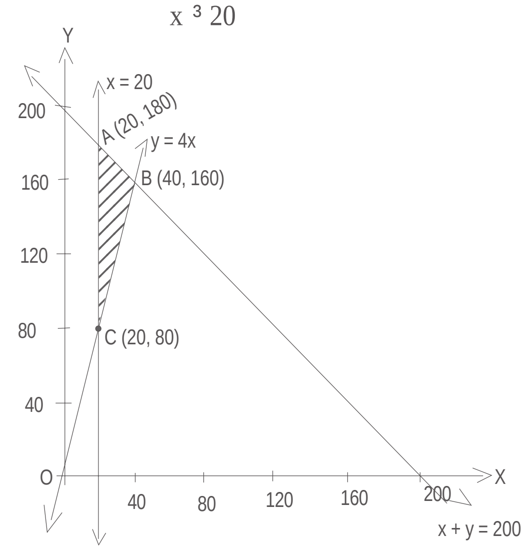
Maximise Profit (P) = $1000x + 600y$

Subject to: $x \geq 0, y \geq 0$

$x + y \leq 200$

$y \geq 4x$ or $4x - y \leq 0$

$x \geq 20$



Getting vertices of feasible region as A (20, 180), B (40, 160), C (20, 80).

Profit at A = Rs. 128000

Profit B = Rs. 136000

Profit at C = Rs. 68000

\ Max profit = Rs. 136000 for 40 executives and 160 economy tickets.

29. Let the events be defined as:

E_1 : Bag A is selected

E_2 : Bag B is selected

E_3 : Bag C is selected

A : A red ball is selected

$$P(E_1) = \frac{1}{6},\ P(E_2) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_3) = \frac{3}{6} = \frac{1}{2}$$

$$P\left(A/E_1\right) = \frac{3}{5},\ P\left(A/E_2\right) = \frac{3}{7} \text{ and}$$

$$P\left(A/E_3\right) = \frac{4}{9}.$$

$$P\left(E_2/A\right) = \frac{P(E_2)\,P\left(A/E_2\right)}{P(E_1)\,P\left(A/E_1\right) + P(E_2)\,P\left(A/E_2\right) + P(E_3)\,P\left(A/E_3\right)}$$

$$= \frac{\frac{1}{3}\cdot\frac{3}{7}}{\frac{1}{6}\cdot\frac{3}{5} + \frac{1}{3}\cdot\frac{3}{7} + \frac{1}{2}\cdot\frac{4}{9}} = \frac{90}{293}.$$

SET II

1. 6
2. 4
3. 27
4. 17. $\left(\because \begin{vmatrix} 2-k & 3 \\ -5 & 1 \end{vmatrix} = 0 \Rightarrow k = 17\right)$
5. 5×2 $\quad$ ($\because$ Order of AB is 2×5)
6. $\sin^{-1}\left(\frac{x}{2}\right)$
7. tan x
8. $-\left(3\hat{i}+2\hat{j}+7\hat{k}\right)$ $\quad \left(\because \overline{AB}+\overline{BC}+\overline{CA} = 0\right)$
9. $\lambda = -9$ $\quad \left(\because \vec{a}.\vec{b} = 0\right)$
10. $\lambda = -3$ $\quad \left(\because \frac{9}{3} = \frac{\lambda}{-1} = \frac{-6}{-2} \Rightarrow \lambda = -3\right)$
11. Let x, y be any two elements of R (domain) then $f(x) = f(y) \Rightarrow 2x^3 - 7 = 2y^3 - 7$

$\Rightarrow x^3 = y^3 \Rightarrow x = y$

So, f is an injective function.

Let y be any element of R (co-domain).

\ $f(x) = y \Rightarrow 2x^3 - 7 = y$

$$\Rightarrow x^3 = \frac{y+7}{2} \Rightarrow x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}}$$

Now for all $y \in R$ (co-domain), there exists

$x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}} \in R$ (domain) such that

$$f(x) = f\left\{\left[\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right]\right\} = 2\left\{\left[\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right]\right\}^3 - 7$$

$$= 2.\frac{y+7}{2} - 7 = y$$

so, f is surjective.

Hence, f is a bijective function.

OR

$$f_og\left(\frac{5}{2}\right) = f\left[g\left(\frac{5}{2}\right)\right] = f(2) = |2| = 2$$

$$g_of\left(-\sqrt{2}\right) = g\left[f\left(-\sqrt{2}\right)\right] = g\left[\left|-\sqrt{2}\right|\right] = g\left[\sqrt{2}\right] = 1$$

12. L. H. S. $= \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$$= \frac{\pi}{4} + \frac{\pi}{2} - \cot^{-1} 2 + \frac{\pi}{2} - \cot^{-1} 3$$

$$= \frac{5\pi}{4} - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)$$

$$= \frac{5\pi}{4} - \left(\tan^{-1}\frac{1}{2} + \tan^{1}\frac{1}{3}\right)$$

$$= \frac{5\pi}{4} - \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}\right) = \frac{5\pi}{4} - \tan^{-1}(1)$$

$$= \frac{5\pi}{4} - \frac{\pi}{4} = \pi = \text{RHS.}$$

13. $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$A^2 - 5A - 14I$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20+0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Premultiplying $A^2 - 5A - 14I = 0$ by A^{-1}, we get

$A^{-1} A^2 - 5A^{-1} A - 14A^{-1} I = 0$

or, $A - 5I - 14A^{-1} = 0$

$$\text{or } A^{-1} = \frac{1}{14}(A - 5I) = \frac{1}{14}\left\{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}\right\}$$

$$= \frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}.$$

14. $f(x) = |(x-3)| \Rightarrow f(x) = \begin{cases} x-3, & \text{if } x \geq 3 \\ -(x-3), & \text{if } x<3 \end{cases}$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} -(x-3) = 0$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x-3) = 0$$

and $f(3) = 3 - 3 = 0$

$$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3).$$

$\therefore$ f(x) is continuous at x = 3.

For differentiability,

$$Lf'(3) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3^-} \frac{-(x-3)-0}{x-3} = -1$$

$$Rf'(3) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3^+} \frac{(x-3)-0}{x-3} = 1$$

$\therefore Lf'(3) \neq Rf'(3)$

so, f(x) is not differentiable at x = 3.

OR

$$\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \tan^{-1} a \text{ ----- (1)}$$

Differentiating (1) w. r. t. x, we get

$$\frac{(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right) - (x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)}{(x^2+y^2)^2} = 0$$

$$\Rightarrow 2x(x^2+y^2) - 2y(x^2+y^2)\frac{dy}{dx} - 2x(x^2-y^2)$$

$$- 2y(x^2-y^2)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}[-2x^2y - 2y^3 - 2x^2y + 2y^3]$$

$$= -2x^3 - 2xy^2 + 2x^3 - 2xy^2$$

$$\Rightarrow \frac{dy}{dx}[-4x^2y] = -4xy^2 \Rightarrow \frac{dy}{dx} = \frac{-4xy^2}{-4x^2y} = \frac{y}{x}$$

15. We know that e^x, sin x and cos x functions are continuous and differentiable everywhere. Product, sum and difference of two continuous functions is again a continuous function, so f is also continuous in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

$$\text{Now, } f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) = 0$$

$$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}}\left(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}\right) = 0$$

$$\Rightarrow \; f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right)$$

$\therefore$ Rolle's theorem is applicable.

$f'(x) = e^x(\sin x - \cos x) + e^x(\cos x + \sin x) = 2e^x \sin x$

$f'(x) = 0 \Rightarrow 2e^x \sin x = 0$

$\Rightarrow \sin x = 0 \Rightarrow x = 0, \pi$

Now $\pi \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\therefore$ The theorem is verified with $x = \pi$

16. Let $x = 25$, $x + \Delta x = 25.2$ so $\Delta x = 0.2$

Let $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{25}} = \frac{1}{10}$ at $x = 25$.

$$dy = \frac{dy}{dx}.\Delta x$$

or $\Delta y = \frac{dy}{dx}.\Delta x = \frac{1}{10} \times 0.2 = 0.02$

$\therefore \sqrt{25.2} = y + \Delta y = 5 + 0.02 = 5.02$

OR

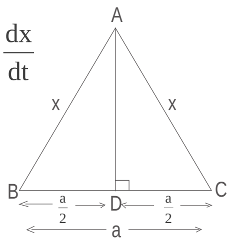
Let A be the area of Δ ABC in which AB = AC = x and BC = a

$$\therefore A = \frac{1}{2} BC \times AD$$

$$= \frac{1}{2} a \sqrt{x^2 - \frac{a^2}{4}} = \frac{a}{4}\sqrt{4x^2 - a^2}$$

$$\frac{dA}{dt} = \frac{a}{4} . \frac{1}{2\sqrt{4x^2 - a^2}} .8x. \frac{dx}{dt}$$

$$= \frac{ax \times 9}{\sqrt{4x^2 - a^2}}$$



$$\therefore \left(\frac{dA}{dt}\right)_{at\ x=a} = \frac{9a.a}{\sqrt{3a^2}} = 3\sqrt{3} \text{ cm}^2\text{/second.}$$

17. $I = \int_{-1}^{1/2} |x \cos(\pi x)| \, dx$

Three cases arise:

Case I : $-1 < x < \frac{-1}{2} \Rightarrow -\pi < \pi x < -\frac{\pi}{2}$

$\Rightarrow \cos \pi x < 0 \Rightarrow x \cos \pi x > 0$

Case II: $-\frac{1}{2} < x < 0 \Rightarrow -\frac{\pi}{2} < \pi x < 0$

$\Rightarrow \cos(\pi x) > 0 \Rightarrow x \cos(\pi x) < 0$

Case III: $0 < x < \frac{1}{2} \Rightarrow 0 < \pi x < \frac{\pi}{2}$

$\Rightarrow \cos \pi x > 0 \Rightarrow x \cos \pi x > 0$

$$\therefore I = \int_{-1}^{-1/2} x \cos \pi x \, dx + \int_{-1/2}^{0} -x \cos \pi x \, dx + \int_{0}^{1/2} x \cos \pi x \, dx$$

$$= \left[\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_{-1}^{-\frac{1}{2}} - \left[\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_{-\frac{1}{2}}^{0}$$

$$+ \left[\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_{0}^{\frac{1}{2}}$$

$$= \left[\left(\frac{1}{2\pi} + 0\right) - \left(0 - \frac{1}{\pi^2}\right)\right] - \left[-\left(\frac{1}{2\pi} + 0\right) + \left(0 + \frac{1}{\pi^2}\right)\right]$$

$$+ \left[-\left(0 + \frac{1}{\pi^2}\right) + \left(\frac{1}{2\pi} + 0\right)\right]$$

$$= \frac{1}{2\pi} + \frac{1}{\pi^2} + \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{1}{2\pi} - \frac{1}{\pi^2} = \frac{3}{2\pi} - \frac{1}{\pi^2}.$$

18. $y e^{\frac{x}{y}} dx = \left(x e^{\frac{x}{y}} + y\right) dy$

$$\Rightarrow \frac{dx}{dy} = \frac{x e^{\frac{x}{y}} + y}{y . e^{\frac{x}{y}}}$$

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y . \frac{dv}{dy}$

$$\therefore\ v + y\frac{dv}{dy} = \frac{vy.e^v + y}{y.e^v}$$

$$\Rightarrow\ y\frac{dv}{dy} = \frac{vye^v + y}{y.e^v} - v = \frac{vye^v + y - vye^v}{y.e^v} = \frac{1}{e^v}$$

$$\Rightarrow\ e^v dv = \frac{dy}{y}$$

Integrating we get $e^v = \log y + \log C = \log Cy$

Substituting $v = \frac{x}{y}$, we get

$$e^{\frac{x}{y}} = \log Cy.$$

19. $(1 + y + x^2y)\,dx + (x + x^3)\,dy = 0$

$\Rightarrow\ x(1 + x^2)\,dy = -[1 + y(1 + x^2)]\,dx$

$$\Rightarrow\ \frac{dy}{dx} = \frac{-1-y\left(1+x^2\right)}{x\left(1+x^2\right)} = \frac{-1}{x}.y - \frac{1}{x\left(1+x^2\right)}$$

$$\text{or}\ \frac{dy}{dx} + \frac{1}{x}.y = -\frac{1}{x\left(1+x^2\right)}$$

$$\therefore\ \text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$\therefore$ The solution is

$$y\,.\,x = -\int \frac{1}{x\left(1+x^2\right)}.x\,dx = -\int \frac{dx}{1+x^2}$$

$= \tan^{-1} x + C$

when $x = 1, y = 0$

$0 = -\tan^{-1}(1) + C \Rightarrow\ C = \pi/4$

$$\therefore\ xy = -\tan^{-1}x + \frac{\pi}{4}.$$

20. $\vec{a}.\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$

$\Rightarrow\ \vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$

$\therefore\ \vec{a}$ is $\perp$ to the plane of $\vec{b}$ and $\vec{c}$.

$\Rightarrow\ \vec{a}$ is parallel to $\vec{b} \times \vec{c}$

Let $\vec{a} = k\left(\vec{b} \times \vec{c}\right)$, where k is a scalar.

$$\therefore\ |\vec{a}| = |\vec{k}|\,|\vec{b} \times \vec{c}|$$

$$= |\vec{k}|\,|\vec{b}|\,|\vec{c}|\sin\frac{\pi}{6}$$

$$\therefore\ 1 = |\vec{k}|\frac{1}{2} \Rightarrow\ |\vec{k}| = 2$$

$\Rightarrow\ k = \pm 2$

$\therefore\ \vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$.

21. Equation of plane passing through $(0, -1, -1)$ is

$a(x - 0) + b(y + 1) + c(z + 1) = 0$ ---- (i)

(i) passes through (4, 5, 1) and (3, 9, 4)

$\Rightarrow\ 4a + 6b + 2c = 0$ or $2a + 3b + c = 0$ ---- (ii)

and $3a + 10b + 5c = 0$ ------ (iii)

From (ii) and (iii), we get

$$\frac{a}{15-10} = \frac{-b}{10-3} = \frac{c}{20-9}$$

$$\Rightarrow\ \frac{a}{5} = \frac{-b}{7} = \frac{c}{11} = k\ (\text{say})$$

$\therefore\ a = 5k, b = -7k, c = 11k$ ---- (iv)

Putting these values of a, b, c in (i), we get

$5kx - 7k(y + 1) + 11k(z + 1) = 0$

or $5x - 7y + 11z + 4 = 0$ ----- (v)

Putting the point $(-4, 4, 4)$ in (v), we get

$-20 - 28 + 44 + 4 = 0$ which is satisfied.

$\therefore$ The given points are co-planar and equation of plane is $5x - 7y + 11z + 4 = 0$.

22. Here $P(H) = \frac{3}{4}$, $P(T) = \frac{1}{4}$

Let X be the random variate, which can take values 0, 1, 2, 3.

P (X = 0) = P (No tails) = P (HHH)

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

P (X = 1) = P (1 Tail) = P (HHT) + P (HTH) + P (THH)

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

$P(X=2) = P(2 \text{ tails}) = P(HTT) + P(THT) + P(TTH)$

$$= \frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{64}$$

$P(X=3) = P(3 \text{ tails}) = P(TTT)$

$$= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$$

Required. probability distribution is

X	0	1	2	3
P(X)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

OR

For a fair coin, $p(H) = \frac{1}{2}$ and $P(T) = \frac{1}{2}$ where H and T denote head and tail respectively.

Let the coin be tossed n times.

$\therefore$ Required probability = 1 – P (all tails)

$$= 1 - \frac{1}{2^n} \text{ ----- (i)}$$

It has to be > 80%.

Total probability = 1

$\therefore$ (i) has to be $> \frac{4}{5}$.

$\Rightarrow 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow n = 3$

$\therefore$ The fair coin has to be tossed 3 times for the desired situation.

23. $$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$, to get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & b^2c & c(a+b)^2 \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Operating $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, to get

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix}$$

$$= (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+b-c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Operating $R_1 \to R_1 - (R_2 + R_3)$ to get

$$\Delta = (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$C_2 \to C_2 + \frac{1}{b}C_1,\ C_3 \to C_3 + \frac{1}{C}C_1$$

$$= (a+b+c)^2\begin{vmatrix} 2bc & 0 & 0 \\ b^2 & a+c & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$

$= (a+b+c)^2\,[2bc\,(a^2 + ac + ab + bc - bc)]$

$= (a+b+c)^2\,(2bc)\,a\,(a+b+c)$

$= (a+b+c)^3\, 2abc$

24. Let the radius of circle be r and side of square be x.

$\therefore$ $2\pi r + 4x = k$ ----- (A)

Let A be the sum of the areas of circle and square.

$\therefore$ $A = \pi r^2 + x^2$

$$= \pi\left[\frac{k-4x}{2\pi}\right]^2 + x^2 \quad \text{[Using (A)]}$$

$$= \pi\left[\frac{k^2+16x^2-8kx}{4\pi^2}\right] + x^2$$

$$= \frac{k^2+16x^2-8kx}{4\pi} + x^2$$

$$\therefore \frac{dA}{dx} = \frac{1}{4\pi}[0 + 32x - 8k] + 2x$$

$= \frac{1}{4\pi}\,[32x - 8k + 8\pi x]$

For optimisation $\frac{dA}{dx} = 0 \Rightarrow (32 + 8\pi x) = 8k$

$\Rightarrow x = \frac{k}{4+\pi}$ ----- (i)

$\therefore \frac{d^2A}{dx^2} = \frac{1}{4\pi}\,[32 + 8\pi] > 0 \Rightarrow$ Minima

Putting the value of x in (A) we get

$2\pi r + 4 \cdot \frac{k}{4+\pi} = k$

$\Rightarrow 2\pi r = k - \frac{4k}{4+\pi} = \frac{\pi k}{4+\pi}$

$2r = \frac{k}{4+\pi}$ ------ (ii)

From (i) and (ii), $x = 2r$

OR

Let P (x, y) be the position of the Helicopter and the position of soldier at A (3, 2).

$\therefore AP = \sqrt{(x-3)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (x^2)^2}$

[$\therefore y = x^2 + 2$ is the equation of curve]

Let $AP^2 = z = (x-3)^2 + x^4$

$\Rightarrow \frac{dz}{dx} = 2(x-3) + 4x^3$

For optimisation $\frac{dz}{dx} = 0 \Rightarrow 2x^3 + x - 3 = 0$

or $(x-1)(2x^2 + 2x + 3) = 0 \Rightarrow x = 1$

[other factor gives no real values]

$\frac{d^2z}{dx^2} = 6x^2 + 1 > 0 \Rightarrow$ Minima

when $x = 1$, $y = x^2 + 2 = 3$

$\therefore$ The required point is (1, 3).

And distance $AP = \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{5}$

25. $\int \frac{1}{\sin x\,(5 - 4\cos x)}\,dx = \int \frac{\sin x}{\sin^2 x\,(5 - 4\cos x)}\,dx$

$= \int \frac{\sin x}{(1-\cos^2 x)(5 - 4\cos x)}\,dx$

$= -\int \frac{dt}{(1-t^2)(5-4t)}$, where $\cos x = t$, $dt = -\sin x\,dx$

$= -\int \frac{dt}{(1-t)(1+t)(5-4t)}$

Let $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t}$

$\Rightarrow 1 = A(1+t)(5-4t) + B(1-t)(5-4t) + C(1-t^2)$ ---- (i)

Putting $t = 1$ in (i) we get $A = \frac{1}{2}$

Putting $t = -1$ in (i) we get $B = \frac{1}{18}$

Putting $t = \frac{5}{4}$ in (i) we get $C = -\frac{16}{9}$

$\therefore I = -\left[\frac{1}{2}\int\frac{dt}{1-t} + \frac{1}{18}\int\frac{dt}{1+t} - \frac{16}{9}\int\frac{dt}{5-4t}\right]$

$= -\left[-\frac{1}{2}\log|1-t| + \frac{1}{18}\log|1+t| - \frac{16}{9\times 4}\log|5-4t|\right] + C$

$= \frac{1}{2}\log|1-\cos x| - \frac{1}{18}\log|1+\cos x| - \frac{4}{9}\log|5-4\cos x| + C$

OR

$I = \int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx = \int\frac{\sqrt{1-\sqrt{x}}\,.\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}\,\sqrt{1-\sqrt{x}}}\,dx$

$= \int\frac{1-\sqrt{x}}{\sqrt{1-x}}\,dx$

$= \int\frac{dx}{\sqrt{1-x}} - \int\frac{\sqrt{x}}{\sqrt{1-x}}\,dx = I_1 - I_2$

$I_1 = \int(1-x)^{-\frac{1}{2}}\,dx$

$= -2\,(1-x)^{\frac{1}{2}} + C_1$ or $-2\sqrt{1-x} + C_1$

$I_2 = \int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

Let $x = \sin^2\theta$, $dx = 2\sin\theta\cos\theta\,d\theta$

$\therefore I_2 = \int \frac{\sin\theta . 2\sin\theta\cos\theta\,d\theta}{\cos\theta} = 2\int \sin^2\theta\,d\theta$

$= \int (1-\cos 2\theta)d\theta = \theta - \frac{\sin 2\theta}{2} = \theta - \sin\theta\cos\theta + C_2$

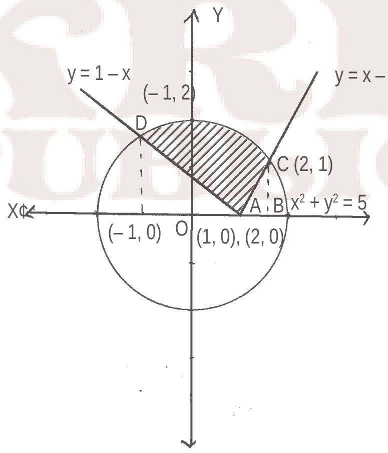
$= \sin^{-1}\sqrt{x} - \sqrt{x}\sqrt{1-x} + C_2$

$\therefore I = -2\sqrt{(1-x)} - \sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} + C$

$= \sqrt{1-x}\left[\sqrt{x} - 2\right] - \sin^{-1}\sqrt{x} + C.$

26. Equations of curves are

$x^2 + y^2 = 5$ and $y = \begin{cases} 1-x, & x<1 \\ x-1, & x>1 \end{cases}$



Points of intersection are C(2, 1) and D (– 1, 2).

Required Area = Area of (EABCDE) – Area of (ADEA) – Area of (ABCA)

$= \int_{-1}^{2}\sqrt{5-x^2}\,dx - \int_{-1}^{1}(1-x)\,dx - \int_{1}^{2}(x-1)\,dx$

$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$

$= \left[\left\{1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right\} - \left\{-\frac{1}{2}\times 2 + \frac{5}{2}\sin^{1}\left(\frac{-1}{\sqrt{5}}\right)\right\}\right]$

$-\left[\left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right)\right] - \left[(2-2) - \left(\frac{1}{2}-1\right)\right]$

$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$

$= -\frac{1}{2} + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} - \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]$

27. Lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are coplanar if

$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

In this case $\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= -2(5-10) + 1(-15+5) + 0 = 10 - 10 = 0$

$\therefore$ Lines are coplanar.

Equation of plane containing this is

$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0 \Rightarrow 5x - 10y + 5z = 0$

or $x - 2y + z = 0$.

28. Let events E_1, E_2, E_3, E_4 and A be defined as follows.

E_1 : Missing card is a diamond
E_2 : Missing card is a spade
E_3 : Missing card is a club
E_4 : Missing card is a heart
A : Drawing two diamond cards

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

$P\left(A/E_1\right) = \frac{12}{51}\times\frac{11}{50}$

$$P\left(A/E_2\right) = P\left(A/E_3\right) = P\left(A/E_4\right) = \frac{13}{51}\times\frac{12}{50}$$

$$P\left(E_4/A\right) = \frac{P(E_4).P\left(A/E_4\right)}{\sum_{i=1}^{4} P(E_i).P\left(A/E_i\right)}$$

$$= \frac{\frac{1}{4}\times\frac{13}{51}\times\frac{12}{50}}{\frac{1}{4}\left[\frac{12\times11+13\times12+13\times12+13\times12}{51.50}\right]}$$

$$= \frac{13\times12}{3\times13\times12+12\times11} = \frac{13}{39+11} = \frac{13}{50}.$$

29. Let x and y be the units taken of Food A and Food B respectively then LPP is,

Minimise $Z = 4x + 3y$

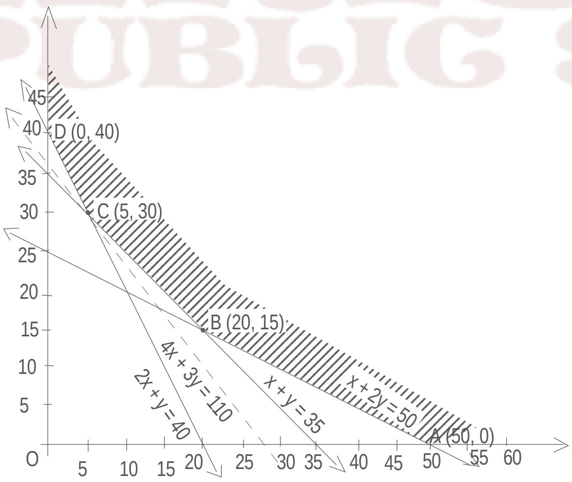
Subject to constraints

$200x + 100y \geq 4000$ or $2x + y \geq 40$

$x + 2y \geq 50$

$40x + 40y \geq 1400$ or $x + y \geq 35$

$x \geq 0, y \geq 0$



The corners of feasible region are

A (50, 0), B (20, 15), C (5, 30), D (0, 40)

Z at A (50, 0) = 200, Z at B (20, 15) = 125,

Z at C (5, 30) = 110, Z at D (0, 40) = 120

$\therefore$ Z is minimum at C (5, 30).

$\therefore$ 5 units of Food A and 30 units of Food B will give the minimum cost which is Rs. 110.

SET III

1. $\frac{5\pi}{6}$

2. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

3. $x = 1$

4. $|A'| = \pm 8$

5. $A^{-1} = A$

6. $\frac{\pi}{180} \cos x^{o}$

7. 2

8. $|\vec{a}\times\vec{b}|$

9. $\frac{\pi}{6}$

10. $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

11. (i) $\forall a \in A$, $|a - a| = 0$ is divisible by 4.

$\therefore$ R is reflexive

(ii) $a, b \in A$, $(a, b) \in R \Rightarrow |a - b|$ is divisible by 4.

$\Rightarrow |b - a|$ is divisible by 4.

$\therefore$ R is symmetric

(iii) $a, b, c \in A$, $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4.

$\therefore$ (a – b) and (b – c) are divisible by 4 and so (a – b) + (b – c) = (a – c) is divisible by 4. Hence $|a - c|$ is divisible by 4 $\Rightarrow (a, c) \in R$.

Hence R is transitive.

Hence R is an equivalence relation from (i), (ii) and (iii).

Set of all elements of A, related to 1 is {1, 5, 9}.

12. Given equation can be written as

$$\tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}\left(\frac{2}{\cos x}\right), 0 < x < \frac{\pi}{2}$$

$$\Rightarrow \frac{2\sin x}{\cos^2 x} = \frac{2}{\cos x} \Rightarrow \tan x = 1$$

$$\Rightarrow x = \frac{\pi}{4}$$

OR

$$\text{LHS} = \tan\frac{1}{2}\,(2\tan^{-1}x + 2\tan^{-1}y)$$

$$= \tan(\tan^{-1}x + \tan^{-1}y) = \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right] = \frac{x+y}{1-xy}$$

13. Given determinant can be written as

$$D = \frac{1}{abc}\begin{vmatrix} -abc & ab(b+c) & ac(b+c) \\ ab(a+c) & -abc & bc(a+c) \\ ac(a+b) & bc(b+a) & -abc \end{vmatrix}$$

$$D = \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ab+ac \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$R_1 \to R_1 + R_2 + R_3$$

$$= (ab+bc+ac)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$D = (ab+bc+ac)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & ab+bc \\ ac+bc & 0 & -(ab+bc+ac) \end{vmatrix}$$

$$C_2 \to C_2 - C_1$$
$$C_3 \to C_3 - C_1$$

$$= (ab+bc+ac)^3$$

14. Putting $x = \cos\alpha$ and $y = \cos\beta$ we get
$\sin\alpha + \sin\beta = a(\cos\alpha - \cos\beta)$

$$\Rightarrow \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = a$$

$$\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right) = -a \Rightarrow \alpha - \beta = 2\cot^{-1}(-a)$$

or $\cos^{-1}x - \cos^{-1}y = 2\cot^{-1}(-a)$

Differentiating we get, $-\dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$$

15. $$\frac{dy}{dx} = m.\left(x+\sqrt{x^2+1}\right)^{m-1}\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{m\left(x+\sqrt{x^2+1}\right)^m}{\sqrt{x^2+1}} = \frac{m.y}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1}.\frac{dy}{dx} = my \quad \text{----- (i)}$$

$$\therefore \sqrt{x^2+1}.\frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2+1}}.\frac{dy}{dx} = m.\frac{dy}{dx}$$

$$\Rightarrow (x^2+1)\frac{d^2y}{dx^2} + x.\frac{dy}{dx} = m\sqrt{x^2+1}\frac{dy}{dx}$$

$$= m.my = m^2y \quad \text{(Using i)}$$

$$\text{or } (x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0.$$

16. $$f(x) = [x^2],\ 1 \le x < 2 \Rightarrow f(x) = \begin{cases} 1, & 1 \le x < \sqrt{2} \\ 2, & \sqrt{2} \le x < \sqrt{3} \\ 3, & \sqrt{3} \le x < 2 \end{cases}$$

At $x = \sqrt{2}$, LHL = 1, RHL = 2

$\therefore\ x = \sqrt{2}$ is a discontinuity of f (x).

At $x = \sqrt{3}$, LHL = 2, RHL = 3

$\therefore\ x = \sqrt{3}$ is also a discontinuity of f (x).

ie., $\sqrt{2}$, $\sqrt{3}$ are two discontinuities in [1, 2)

OR

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $z = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

Put $x = \sin\theta$ weo get
$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$ and $z = 2\tan^{-1}x$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \text{and} \quad \frac{dz}{dx} = \frac{2}{1+x^2}$$

$$\Rightarrow \quad \frac{dy}{dz} = \frac{1+x^2}{\sqrt{1-x^2}}.$$

17. $I = \int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$

$$= \frac{1}{\sin(b-a)}\int \frac{\sin[(x-a)-(x-b)]}{\cos(x-a)\cos(x-b)}\,dx$$

$$= \frac{1}{\sin(b-a)}\int\left[\frac{\sin(x-a)\cos(x-b)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-a)\sin(x-b)}{\cos(x-a)\cos(x-b)}\right]dx$$

$$= \frac{1}{\sin(b-a)}\int[\tan(x-a)-\tan(x-b)]\,dx$$

$$= \frac{1}{\sin(b-a)}.[\log|\sec(x-a)| - \log|\sec(x-b)|] + C$$

OR

$$I = \int (\log x)^2 .x\,dx = (\log x)^2.\frac{x^2}{2} - \int 2.\frac{\log x}{x}.\frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}.(\log x)^2 - \log x.\frac{x^2}{2} + \int \frac{1}{x}.\frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}.(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

$$= \frac{x^2}{2}.\left[(\log x)^2 - \log x + \frac{1}{2}\right] + C.$$

18. $\frac{x}{x^3-1} = \frac{x}{(x-1)(x^2+x+1)}$

$$= \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$\Rightarrow$ x = A $(x^2 + x + 1)$ + (Bx + C) (x – 1)

$\Rightarrow$ A + B = 0, A – B + C = 1 and A – C = 0

$\Rightarrow$ $A = \frac{1}{3}, C = -\frac{1}{3}, C = \frac{1}{3}$

$$\therefore\ I = \int \frac{x}{x^3-1}\,dx = \frac{1}{3}\int\frac{1}{x-1}\,dx - \frac{1}{3}\int\frac{x-1}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\int\frac{2x+1}{x^2+x+1}\,dx + \frac{1}{2}\int\frac{1}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)+\left(\sqrt{\frac{3}{2}}\right)^2}\,dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

19. $I = \int_0^{\pi}\frac{x\,dx}{4-\cos^2 x} = \int_0^{\pi}\frac{(\pi-x)\,dx}{4-\cos^2(\pi-x)}$

$$= \int_0^{\pi}\frac{(\pi-x)\,dx}{4-\cos^2 x}$$

$$\therefore\ 2I = \pi\int_0^{\pi}\frac{1}{4-\cos^2 x}\,dx = 2\pi\int_0^{\pi/2}\frac{\sec^2 x}{4-\tan^2 x+3}\,dx$$

$$I = \frac{\pi}{4}\int_0^{\infty}\frac{dt}{t^2+3/4}$$

Put tan x = t
$Sec^2x\,dx = dt$

$$\Rightarrow\ I = \frac{\pi}{4}.\frac{2}{\sqrt{3}}\tan^{-1}\left.\frac{2t}{\sqrt{3}}\right]_0^{\infty}$$

$$I = \frac{\pi}{2\sqrt{3}}.\frac{\pi}{2} = \frac{\pi^2}{4\sqrt{3}}$$

20. Let the required vector be $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore\ \vec{a}.(\hat{i}-3\hat{k}) = 0 \Rightarrow x-3z=0$ ----- (i)

$\vec{a}.(\hat{i}-2\hat{k}) = 5 \Rightarrow x-2z=5$ ---- (ii)

$\vec{a}.(\hat{i}+j+4\hat{k}) = 8 \Rightarrow x+y+4z=8$ ---- (iii)

Solving (i) and (ii) we get x = 15, z = 5

Putting in (iii) we get y = – 27

$\vec{a} = 15\hat{i}-27\hat{j}+5\hat{k}$.

21. Here $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$ and

$\vec{n} = (2-1)\hat{i}+(3-4)\hat{j}+(5-2)\hat{k} = \hat{i}-\hat{j}+3\hat{k}$

$\therefore$ Equation of plane is $\vec{r}.(\hat{i}-\hat{j}+3\hat{k})$

$= (\hat{i}+2\hat{j}+\hat{k}).(\hat{i}-\hat{j}+3\hat{k}) = 2$

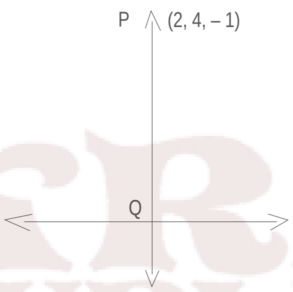
or $x - y + 3z - 2 = 0$

Distance from origin = $\dfrac{2}{\sqrt{1+1+9}} = \dfrac{2}{\sqrt{11}}$

or $\dfrac{2\sqrt{11}}{11}$ units

OR

Any point on the given line is $(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$ for some value of λ, this point is Q, such that PQ is $\perp$ to the line.



$\dfrac{x+5}{1} = \dfrac{y+3}{4} = \dfrac{z+6}{-9}$

$\Rightarrow (\lambda - 7)\,1 + (4\lambda - 7)\,4 + (-9\lambda + 7)(-9) = 0$

$\Rightarrow \lambda = 1$

$\therefore$ Q is $(-4, 1, -3)$ and equation of line PQ is

$\dfrac{x-2}{6} = \dfrac{y-4}{3} = \dfrac{z+1}{2}$

22. Getting P (odd number) = $\dfrac{1}{3}$,

P (even number) = $\dfrac{2}{3}$.

Let X be the random variable "getting an even number".

X	0	1	2	3
P (X)	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$
X . P (X)	0	$\frac{6}{27}$	$\frac{24}{27}$	$\frac{24}{27}$

Mean = $\Sigma X\,P(X) = \dfrac{54}{27} = 2$.

23. Given equation can be written as

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \text{ or } A . X = B$$

$|A| = 1(4) + 1(5) + 1(1) = 10 \neq 0 \; \therefore \; X = A^{-1} . B$

Cofactors are:

$A_{11} = 4,\quad A_{12} = -5,\; A_{13} = 1$

$A_{21} = 2,\quad A_{22} = 0,\; A_{23} = -2$

$A_{31} = 2,\quad A_{32} = 5,\; A_{33} = 3$

$$\therefore A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

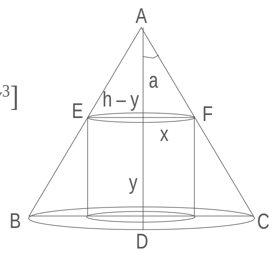
$\Rightarrow x = \dfrac{1}{2}, y = -1, z = 1.$

24. Let the radius of inscribed cylinder be x and its height be y.

$\therefore$ Volume (V) = $\pi x^2 y$

$= \pi (h - y)^2 \tan^2\alpha . y$

$= \pi \tan^2\alpha\,[h^2 y - 2hy^2 + y^3]$



$\dfrac{dV}{dy} = \pi \tan^2\alpha\,[h^2 - 4hy + 3y^2]$

$\dfrac{dv}{dy} = 0 \Rightarrow 3y^2 - 4hy + h^2 = 0$

$\Rightarrow 3(y - h)(3y - h) = 0 \Rightarrow y = h, y = h/3$

Since $y = h$ is not possible $\therefore$ $y = h/3$ is the only point.

$$\frac{d^2V}{dy^2} = 6y - 4h = 6\left(h/3\right) - 4h = -2h < 0$$

$$\therefore\ y = \frac{h}{3} \text{ is a maxima}$$

$$= \frac{4}{27}\pi h^3 \tan^2\alpha$$

OR

$$\frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta = a\theta\sin\theta$$

$\Rightarrow \dfrac{dy}{dx} = \tan\theta$ $\therefore$ slope of normal $= -\cot\theta$

$\therefore$ Equation of normal is

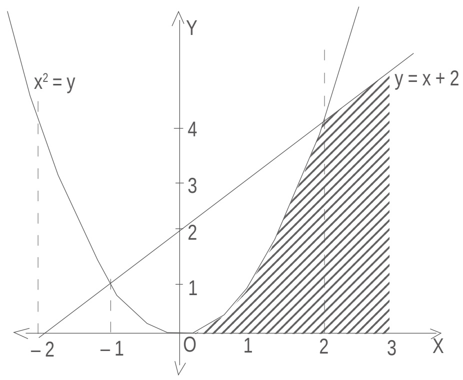
$$y - a(\sin\theta - \theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}[x - a(\cos\theta + \theta\sin\theta)]$$

Simplifying we get $x\cos\theta + y\sin\theta - a = 0$

Length of perpendicular from origin

$$= \frac{|a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| \text{ (constant).}$$

25. From figure we get points of intersection as $x = -1, x = 2$



$$\text{Required area} = \int_0^2 x^2\,dx + \int_2^3 (x+2)\,dx$$

$$\left[\frac{x^3}{3}\right]_0^2 + \left[\frac{(x+2)^2}{2}\right]_2^3 = \frac{8}{3} + \frac{25}{2} - 8 = \frac{43}{6} \text{ sq. u.}$$

26. Given differential equation can be written as

$$\left(xy\frac{dy}{dx} - y^2\right)\sin\left(\frac{y}{x}\right) = \left(xy + x^2\frac{dy}{dx}\right)\cos\left(y/x\right) \text{ ----- (i)}$$

Putting $\dfrac{y}{x} = v$ or $y = vx$ gives $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ (i) become $v\sin v\left(v + x\dfrac{dv}{dx}\right) - v^2\sin v$

$$= v\cos v + \left(v + x\frac{dv}{dx}\right)\cos v$$

$\Rightarrow (vx\sin v - x\cos v)\dfrac{dv}{dx} = 2v\cos v$

$\Rightarrow -\displaystyle\int \frac{v\sin v - \cos v}{v\cos v}\,dv = -\int \frac{2}{x}\,dx$

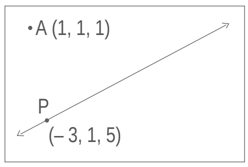
$\Rightarrow \log|v\cos v| = -2\log x + \log C$

$\Rightarrow x^2 . v . \cos v = C \Rightarrow xy\cos\dfrac{y}{x} = C$

$x = 3, y = \pi$ gives $C = \dfrac{3\pi}{2}$

$\therefore$ solution is $2xy\cos\dfrac{y}{x} = 3\pi$

27. Let the given point be A (1, 1, 1) and the point on the line is P (– 3, 1, 5).



$$\overline{AP} = -4\hat{i} + 4\hat{k}$$

$\therefore$ The vector $\perp$ to the plane is

$$\left(-4\hat{i} + 4\hat{k}\right)\times\left(3\hat{i} - \hat{j} - 5\hat{k}\right)$$

$$= 4\hat{i} - 8\hat{j} + 4\hat{k} \text{ or } \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Equation of plane is

$$\vec{r}.\left(\hat{i} - 2\hat{j} + \hat{k}\right) = \left(\hat{i} + \hat{j} + \hat{k}\right).\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0 \text{ ----- (i)}$$

or $x - 2y + z = 0$

Now, since $\left(\hat{i}-2\hat{j}-5\hat{k}\right).\left(\hat{i}-2\hat{j}+\hat{k}\right)=1+4-5=0$

$\therefore$ The line $\vec{r} = \left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}-5\hat{k}\right)$ is parallel to the plane.

Also, the point (– 1, 2, 5) satisfies the equation of plane as (– 1 – 4 + 5) = 0 $\Rightarrow$ point lies on plane.

Hence the plane contains the line.

28. Let x be the number of units of A and y be the number of units of B, which are produced.

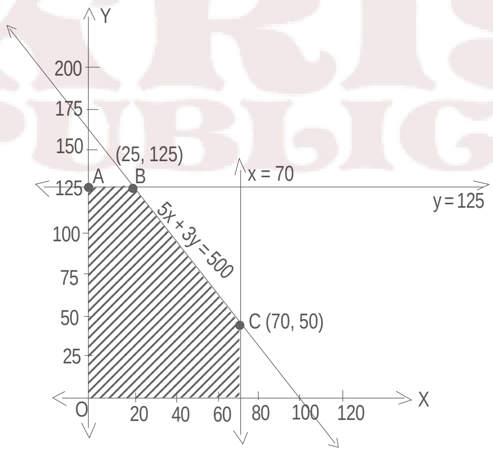
$\therefore$ LPP is Maximise Z = 20x + 15y

Subject to $5x + 3y \le 500$

$x \le 70$

$y \le 125$

$x \ge 0,\ y \ge 0$



Getting vertices of feasible region as:

A (0, 125), B (25, 125), C (70, 50), D (70, 0)

Maximum profit = Rs. 2375 at B

$\therefore$ Number of units of A = 25

Number of units of B = 125

29. Let the events are defined as:

E_1 : 2 white balls are transferred from A to B

E_2 : 2 black balls are transferred

E_3 : 1 white and 1 black ball is transferred

A : 1 black ball is drawn from B

$$P(E_1) = \frac{4C_2}{7C_2} = \frac{4.3}{7.6} = \frac{2}{7},\ P(E_2) = \frac{3C_2}{7C_2} = \frac{3.2}{7.6} = \frac{1}{7},$$

$$P(E_3) = \frac{4C_1 . 3C_1}{7C_2} = \frac{4}{7}$$

$$P(A/E_1) = \frac{2}{6} = \frac{1}{3},\ P(A/E_2) = \frac{4}{6} = \frac{2}{3},$$

$$P(A/E_3) = \frac{3}{6} = \frac{1}{2}$$

$$P(E_3/A) = \frac{P(E_3).P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{4}{7}\times\frac{1}{2}}{\frac{2}{7}.\frac{1}{2}+\frac{1}{7}.\frac{2}{3}+\frac{4}{7}.\frac{1}{2}} = \frac{3}{5}$$

OR

$$P(\text{answer is true}) = \frac{1}{2}$$

$$P(\text{answer is false}) = \frac{1}{2}$$

P (at most 7 correct) = 1 – {P (8) + P (9) + P (10)}

where P (8) etc means probability of 8 correct answers

$$= 1 - \left\{ {}^{10}C_8\left(\frac{1}{2}\right)^8.\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10} \right\}$$

$$= 1 - \{{}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0\}\left(\frac{1}{2}\right)^{10}$$

$$= 1 - \{45 + 10 + 1\}\frac{1}{1024}$$

$$= 1 - \frac{56}{1024} = 1 - \frac{7}{128} = \frac{121}{128}$$

VALUE BASED PROBLEMS

MATHEMATICS

CLASS-XII

RELATIONS AND FUNCTIONS

Q.1 Prove that f: R → R is a bijection given by f(x) = x^3+3. Find $f^{-1}(x)$. Does the truthfulness and honesty may have any relation?

Q.2 Set $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ when $a_i^{'s}$ and $bi^{'s}$ Are school going students. Define a relation from a set A to set B by x R y iff y is a true friend of x.
If R= $\{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$
Is R a bijective function? Do you think true friendship important in life? How?

Q.3 If **h** denotes the number of honest people and p denotes the number of punctual people and a relation between honest people and punctual people is given as h = p+5. If **P** denotes the number of people who progress in life and a relation between number of people who progress and honest people is given as P = (h/8) +5. Find the relation between number of people who progress in life and punctual people. How does the punctuality important in the progress of life?

Q.4 let A be the set of all students of class XII in a school and R be the relation, having the same sex on A, and then prove that R is an equivalence relation. Do you think, co-education may be helpful in child development and why?

MATRICES & DETERMINANTS

Q.5 Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. it is found that the shopkeepers A, B, C are using (20,30,40) , (30,40,20,) , (40,20,30) polythene , handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent Rs.250, Rs.220 & Rs.200 on these carry bags respectively .Find the cost of each carry bags using matrices. Keeping in mind the social & environmental conditions, which shopkeeper is better? & why?

Q.6 In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways; telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, and 5000), (3000, 1000, 10000) and (2000, 1500, 4000), respectively. The party paid Rs. 3700, Rs.7200, and Rs.4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

Q.7 A trust fund has Rs. 30,000 is to be invested in two different types of bonds. The first bond pays 5% interest per annum which will be given to orphanage and second bond pays7% interest per annum which will be given to an N.G.O. cancer aid society. Using matrix multiplication, determine how to divide Rs 30,000 among two types of Bonds if the trust fund obtains an annual total interest of Rs. 1800. What are the values reflected in the question.

Q.8 Using matrix method solve the following system of equations

$x + 2y + z = 7$

$x – y + z =4$

$x + 3y +2z = 10$

If X represents the no. of persons who take food at home. Y represents the no. of parsons who take junk food in market and z represent the no. of persons who take food at hotel.

Which way of taking food you prefer and way?

Q.9 A school has to reward the students participating in co-curricular activities (Category I) and with 100% attendance (Category II) brave students (Category III) in a function. The sum of the numbers of all the three category students is 6. If we multiply the number of category III by 2 and added to the number of category I to the result, we get 7. By adding second and third category would to three times the first category we get 12.Form the matrix equation and solve it.

Q.10 for keeping Fit **X** people believes in morning walk, **Y** people believe in yoga and Z people join Gym. Total no of people are 70.further 20% 30% and 40% people are suffering from any disease who believe in morning walk, yoga and GYM respectively. Total no. of such people is 21. If morning walk cost Rs 0 Yoga cost Rs 500/month and GYM cost Rs 400/ month and total expenditure is Rs 23000.

(i) Formulate a matrix problem.

(ii) Calculate the no. of each type of people.

(iii) Why exercise is important for health?

Q.11 An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is for saving girl child from the cruel parents who don’t want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is Rs 700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is Rs 1200 crores. Find the amount spent on each Scheme using matrices? What is the importance of saving girl child from the cruel parents who don’t want girl child and get the abortion before her birth?

Q.12 There are three families. First family consists of 2 male members, 4 female members and 3 children. Second family consists of 3 male members, 3 female members and 2 children. Third family consists of 2 male members, 2 female members and 5 children. Male member earns Rs 500 per day and spends Rs 300 per day. Female member earns Rs 400 per day and spends Rs 250 per day child member spends Rs 40 per day. Find the money each family saves per day using matrices? What is the necessity of saving in the family?

CONTINUITY AND DIFFERENTIABILITY

Q.13 A car driver is driving a car on the dangerous path given by

$$f(x) = \begin{cases} \frac{1-x^m}{1-x}, & x \neq 1 \\ m-1, & x = 1 \end{cases} \quad m \in N$$

Find the dangerous point (point of discontinuity) on the path. Whether the driver should pass that point or not? Justify your answers.

APPLICATION OF DERIVATIVES

Q.14 A car parking company has 500 subscribers and collects fixed charges of Rs.300 per subscriber per month. The company proposes to increase the monthly subscription and it is believed that for every increase of Re.1, one subscriber will discontinue the service. What increase will bring maximum income of the company? What values are driven by this problem?

Q.15 Check whether the function $f(x) = x^{100} + \sin x - 1$ is strictly increasing or strictly decreasing or none of both on $(-1,1)$. Should the nature of a man be like this function? Justify your answers.

Q.16 If $y = x^4 - \frac{x^3}{3}$, when x denotes the number of hours worked and y denotes the amount (in Rupees) earned. Then find the value of x (in interval) for which the income remains increasing? Explain the importance of earning in life?

Q.17 If performance of the students 'y' depends on the number of hours 'x' of hard work done per day is given by the relation.

$$y = 4x - \frac{x^2}{2}$$

Find the number of hours, the students work to have the best performance.

'Hours of hard work are necessary for success' Justify.

Q.18 A farmer wants to construct a circular well and a square garden in his field. He wants to keep sum of their perimeters fixed. Then prove that the sum of their areas is least when the side of square garden is double the radius of the circular well.

Do you think good planning can save energy, time and money?

Q.19 Profit function of a company is given as $P(x) = \frac{24x}{5} - \frac{x^2}{100} - 500$ where x is the number of units produced. What is the maximum profit of the company? Company feels its social responsibility and decided to contribute 10% of his profit for the orphanage. What is the amount contributed by the company for the charity? Justify that every company should do it.

Q.20 In a competition a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15cm. Also write any three values/life skill reflected in this question.

Q.21 In a kite festival, a kite is at a height of 120m and 130m string is out. If the kite is moving horizontally at the rate of 5.2m/sec, find the rate at which the string is being pulled out at that instant. How a festival enhance national integration.

Q.22 An expensive square piece of golden color board of side 24 centimeters. is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box. What should be the side of the square piece to be cut from each corner of the board to hold maximum volume and minimize the wastage? What is the importance of minimizing the wastage in utilizing the resources?

Q.23 A student is given card board of area 27 square centimeters. He wishes to form a box with square base to have maximum capacity and no wastage of the board. What are the dimensions of the box so formed? Do you agree that students don't utilize the resources properly? Justify.

INTEGRATION

Q.24 Evaluate, $\int \frac{x^3 - x + 1}{x^2 + 1} dx$, Discuss the importance of integration (unity) in life.

APPLICATIONS OF INTEGRALS

Q.25 A farmer has a piece of land. He wishes to divide equally in his two sons to maintain peace and harmony in the family. If his land is denoted by area bounded by curve $y^2 = 4x$ and $x = 4$ and to divide the area equally he draws a line $x = a$ .what is the value of a?
What is the importance of equality among the people?

Q.26 A circular Olympic gold medal has a radius 2cm and taking the centre at the origin, Find its area by method of integration. What is the importance of Olympic Games for a sportsman and why?
Olympic game is a supreme platform for a sportsman. In Olympic Games all countries of the world participate and try their best and make their country proud.

Q.27 A poor deceased farmer has agriculture land bounded by the curve y= $\cos x$, between x = 0 and x=2 π. He has two sons. Now they want to distribute this land in three parts (As already partitioned).Find the area of each part. Which parts should be given to the farmer & why? Justify your answer.

Q.28 If a triangular field is bounded by the lines x+2y = 2, y-x = 1 and 2x+y = 7Using integration compute the area of the field
(i)If in each square unit area 4 trees may be planted. Find the number of trees can be planted In the field.

(ii)Why plantation of trees is necessary?

DIFFERENTIAL EQUATIONS

Q.29 Solve the differential equation $(x + 2y^2)\ \frac{dy}{dx} = y$

Given that when x=2, y=1.If x denotes the $\%\ of$ people who are Polite and y denotes the % of people who are intelligent. Find x when $y = 2\%$.
A polite child is always liked by all in society. Do you agree? Justify.

Q.30 $\frac{dy}{dx} + \frac{y}{x}$=0, where x denotes the percentage population living in a city & y denotes the area for living a healthy life of population. Find the particular Solution when $x = 100, y = 1.$
Is higher density of population is harmful? Justify yours answer.

VECTORS & 3-DIMENSIONAL GEOMETRY

Q.31 considering the earth as a plane having equation $5x + 9y - 10z + 138 = 0$, A monument is standing vertically such that its peak is at the point (1, 2, -3). Find the height of the monument.
How can we save our monument?

Q.32 Let the point p (5, 9, 3) lies on the top of Qutub Minar, Delhi. Find the image of the point on the line
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$
Do you think that the conservation of monuments is important and why?

Q.33 Two bikers are running at the Speed more than allowed speed on the road along the Lines

$\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda\,(3\hat{\imath} - \hat{\jmath}\,)$ **and**

$\vec{r} = 4\,\hat{\imath} - \hat{k} + \mu\,(2\hat{\imath} + 3\,\hat{k}\,)$

Using Shortest distance formula check whether they meet to an accident or not? While driving should driver maintain the speed limit as allowed. Justify?

LINEAR PROGRAMMING PROBLEMS

Q.34 A dietician wishes to mix two types of food in such a way that the vitamin content of the mixture contain at least 8 unit of vitamin A and 10 unit of vitamin C. Food I contains 2unit/kg

of vitamin A and 1unit/kg of vitamin C, while food II contains I unit/kg of vitamin A and 2unit/kg of vitamin C. It cost Rs.5.00 per kg to purchase food I and Rs.7.00 per kg to produce food II. Determine the minimum cost of the mixture. Formulate the LPP and solve it.
Why a person should take balanced food?

Q.35 A farmer has a supply of chemical fertilizers of type 'A' which contains 10% nitrogen and 6% phosphoric acid and type 'B' contains 5% of nitrogen and 10% of phosphoric acid. After soil testing it is found that at least 7kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizers of type A and type B cost Rs.5 and Rs.8 per kilograms respectively. Using L .P.P, find how many kgs of each type of fertilizers should be bought to meet the requirement and cost be minimum solve the problem graphically. What are the side effects of using excessive fertilizers?

Q.36 If a class XII student aged 17 years, rides his motor cycle at 40km/hr, the petrol cost is Rs. 2 per km. If he rides at a speed of 70km/hr, the petrol cost increases Rs.7per km. He has Rs.100 to spend on petrol and wishes to cover the maximum distance within one hour.

1. Express this as an L .P.P and solve graphically.
2. What is benefit of driving at an economical speed?
3. Should a child below 18years be allowed to drive a motorcycle? Give reasons.

Q.37 Vikas has been given two lists of problems from his mathematics teacher with the instructions to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. Vikas knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way how many problems from each list shall he do to maximize his marks? What is the importance of time management for students?

Q.38 An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but there are 45000 bottles into which either of the medicine can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B and there are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan his production to cover maximum patients? How can you help others in case of natural disaster?

PROBABILITY

Q.39 Probability of winning when batting coach A and bowling coach B working independently are ½ and ⅓ respectively. If both try for the win independently find the probability that there is a win. Will the independently working may be effective? And why?

Q.40 A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time. What values are driven by this question?

Q.41 A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin tossed twice find the probability distribution and mean of numbers of tails. Is this a good tendency? Justify your answer.

Q.42 A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white?
Do you think that speaking truth is always good?

Q.43 A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point. Do you think drinking habit can ruin one's family life?

Q.44 If group A contains the students who try to solve the problem by knowledge, Group B contains the students who guess to solve the problem Group C contains the students who give answer by cheating. If n (A) = 20, n (B) = 15, n(C) = 10, 2 Students are selected at random. Find the probability that they are from group c. Do you think that cheating habit spoils the career?

Q.45 In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance? Also state the factors which affect the result of a student in the examination.

Q.46 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six. Write any three benefits of speaking the truth.

Q.47 There are 20 People in a group. Out of them 7 people are non –vegetarian, 2 people are selected randomly. Write the probability distribution of non–vegetarian people. Explain whether you would like to be vegetarian or non- vegetarian and why? Also keeping life of animals in mind how would you promote a person to be vegetarian?

Q.48 Two third of the students in a class are sincere about their study and rest are careless Probability of passing in examination are 0.7 and 0.2 for sincere and careless students respectively, A Student is chosen and is found to be passed what is the probability that he/she was sincere. Explain the importance of sincerity for a student.

Q.49 A company has two plants of scooter manufacturing. Plant I manufacture 70% Scooter and plant II manufactures 30%. At plant I 80% of the scooter's are maintaining pollution norms and in plant II 90% of the scooter maintaining Pollution norms. A Scooter is chosen at random and is found to be fit on pollution norms. What is the probability that it has come from plant II. What is importance of pollution norms for a vehicle?

Q.50 A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job. Find the probability distribution for selection of persons other than their relatives.
If the chairman is biased than which value of life will be demolished?

Q.51 A manufacturer has three machine operators A (skilled) B (Semi- skilled) and C (non-skilled).The first operator A Produces 1% defective items where as the other two operators B and C produces 5% and 7 % defective items respectively. A is on the job for 50% of time B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced what is the probability that it was produced by B? What is the value of skill?

Q.52 In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families feel both children are equal. What is the importance in the society to develop the feeling that both children are equal?

Q.53 In a group of 200 people, 50% believe in that anger and violence will ruin the country, 30% do not believe in that anger and violence will ruin the country and 20% are not sure about anything. If 3 people are selected at random find the probability that 2 people believe and 1 does not believe that anger and violence will ruin the country. How do you consider that anger and violence will ruin the country?

Q.54 In a group of students, 200 attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers. The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with help of peers are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he study himself with help of peers. What type of study can be considered for the success in life and why?

RELATIONS AND FUNCTION

Ans.1 $f^{-1}(x) = (x-3)^{\frac{1}{3}}$,

Truthfulness and honesty among people may have the bijective (one-one onto) relation as people who are honest usually truthful and vice versa.

Ans.2 Neither one-one nor onto hence not bijective
Yes, true friendship makes life easier.

Ans.3 $P = \frac{p+45}{8}$,
Punctuality develops discipline in life and hence progressive in life.

Ans.4 The relation R is reflexive, symmetric and transitive .Co-education is very helpful because it leads to the balanced development of the children and in future they become good citizens.

MATRICES & DETERMINANTS

Ans.5 [Polythene=Re.1]
[Handmade bag = Rs.5]
[Newspaper's envelop=Rs.2]
Shopkeeper A is better for environmental conditions. As he is using least no of polythene.
Shopkeeper B is better for social conditions as he is using handmade bags (Prepared by prisoners).

Ans.6 Cost per Contact:
Telephone = Rs0.40
House calls = Re1.00
Letters = Rs0.50
Telephone is better as it is cheap.

Ans.7 Rs.15000 each type of bond.
(i) Charity.
(ii) Helping orphans or poor people.
(iii)Awareness about diseases.

Ans.8 X = 3, Y =1, Z = 2
Food taken at home is always the best way.

Ans.9 x+y+z=6, x+2z=7, 3x+y+z=12 where x,y,z represent the number of students in categories I,II,III respectively.
X=3, y=1, z=2
Participating in co-curricular activities is very important. It is very essential for all round development.

Ans.10 (i) x+y+z=70, 2x+3y+4z=210, 5y+4z=230
(ii) x=20, y=30, z=20
(iii) Exercise keeps fit and healthy to a person.

Ans.11 Rs300crores, Rs200crores and Rs100 crores
(i) Our In country, male population is more than female population.
(ii) It is essential for a human being to save the life of all.

Ans.12 Rs880, Rs970, Rs 500. Saving is necessary for each family as in case of emergency our saving in good time helps us to survive in bad time.

CONTINUITY AND DIFFERENTIABILITY

Ans.13 [Point $of\ Discontinuity \quad x = 1$]
No, because
Life is precious. Or
Drive carefully.

APPLICATION OF DERIVATIVES

Ans.14 Increase of Rs.100 monthly subscription for Max. Income of the company.
1. The sharing (2-3 persons on the same route) will be promoted.
2. Decrease pollution
3. Decrease vehicle density on road.
4. Saving of energy.

Ans.15 [Neither strictly increasing nor strictly decreasing].
Yes, because strictness in not always good in life.

Ans.16 $x \in \left(\frac{1}{4}, \infty\right)$
To support the family, regular increasing income is must.

Ans.17 4 hours per day. By hard work, we can create skill in using the things Learnt by us. So we Don't make mistake in the competition when the things are asked.

Ans.18 Yes, every work done in a planned way proves to be more fruitful.
If a student makes a planning for his studies he can do wonders.

Ans.19 Maximum profit = Rs76 when x=240.
Yes it is good for society

Ans.20 15/2π Cm. /Sec.
(i) Bravery

(ii) Awareness about child labour
(iii) Right of a child

Ans.21 4.8m/sec.
In a festival many people participated with full happiness and share their lives and enjoy it.

Ans.22 4 centimeters. As our country is still developing and most of the Indian people are from the middle class, so we should utilize our resources in proper way. Students should buy only those books which they feel really important. Instead of buying books for only one or two chapters. They should borrow it from the library.

Ans.23 length of square base is 3 centimeters and height of the box is 1.5 centimeters. Yes, I agree that students don't utilize the resources properly. They get various notes photocopies and waste one side of the paper. Whereas other side of paper can be utilized for making comments on those notes.

INTEGRATION

Ans.24 $\frac{x^2}{2}$ - log| x^2+1|+ tan $^{-1}$ x + C

1. United we stand, divided we fall.
2. Union is strength.

APPLICATIONS OF INTEGRALS

Ans.25 $a = (16)^{1/3}$. Equality helps to maintain peace and harmony in all aspect of society

Ans.26 4πcm^2

Ans.27 1, 2, 1
1. Respect the parents
2. Help the elders (parents)

Ans.28 Area of the field= 6 Sq. unit
(i) 24 trees
(ii) Plants provide us oxygen and play major role in rain, so plantation is essential for all human beings.

DIFFERENTIAL EQUATIONS

Ans.29 $x = 2y^2$, 8. Yes polite child has a peaceful mind and peaceful mind grasps the ideas easily and understand the complicated concept

Ans.30 [$xy = 100$]
Yes, as the population increases area for living decreases, that is very harmful for us.

VECTORS & 3-DIMENSIONAL GEOMETRY

Ans.31 (i) $\frac{191}{\sqrt{206}}$ Units

(ii) We should not harm any monument.

(iii) We should not write anything on it.

(iv) We should respect our national heritage.

Ans.32 The point of image is (3, 5, 7)

Conservation of monuments is very important because it is a part of our history and their contribution.

Ans.33 S.D =0, this means they meet to an accident.

If a driver follow speed limit there will be minimum chance of accident.

LINEAR PROGRAMMING PROBLEMS

Ans.34 Minimum cost = Rs. 38.00

x=2, y=4

Balanced diet keeps fit, healthy and disease free life to a person.

Ans.35 Type A fertilizers = 50 kg, Type B = 40 kg. Minimum cost =Rs. 570/-

infertility of land.

Excessive use of fertilizers can spoil the quality of crop also it may cause.

Ans.36 1. Max. Z= x + y,

$\frac{x}{40}+\frac{y}{70} \leq 1,$

$2x+7y \leq 100,$

$X \geq 0,\ y \geq 0$

Where x & y represents the distance travelled by the speed of 40km/hr & 70 km/h respectively.

1. X=1560/41Km., y= 140/41Km.
2. It Saves petrol. It saves money.
3. No because according to the law driving license is issued when a person is above the 18 years of age.

Ans.37 20 problems from first list and 80 problems from second list. Students who divide the time for each subject per day according to their need don't feel burden of any subject before the examination.

Ans.38 10500 bottles of medicine A and 34500 bottles of medicine B and they can cover 325500 patients. We should not get panic and should not create panic in case of natural disaster. Must have the helpline numbers of government agencies and NGO working in case of Natural Disaster.

PROBABILITY

Ans.39 $\left[\frac{2}{3}\right]$

1. Chances of success increase when ideas flow independently.
2. Hard work pays the fruits.

Ans.40 [0.488]

Peace is better than strike. As the probability of completion of job on time if there is strike is less then ½.

Ans.41

x	0	1	2
P(x)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Mean = $\frac{1}{2}$

1. No, it may be good once or twice but not forever.
2. Honesty pays in a long run.

Ans.42 $\frac{10}{13}$, speaking truth pays in the long run. Sometimes lie told for a good cause is not bad.

Ans.43 $126 \times (0.6)^4(0.4)^4$ or $126(0.24)^4$

Yes, addiction of wine or smoking is definitely harmful for a person and its family.

Ans.44 (i) $\frac{1}{22}$

(ii) Yes, because a cheater finds it to do any work independently. But it is harmful in long run.

Ans.45 3/4

Factors :-(i) Regular study
(ii) Hard work
(iii) Good memory
(iv) Well time management
(v) Writing skills

Ans.46 3/8

(i) It gives positive thinking &satisfaction
(ii) Everyone loves it.
(iii) It is good life skill

Ans.47

x	0	1	2
$p(x)$	$\frac{156}{380}$	$\frac{192}{380}$	$\frac{42}{380}$

I would like to be a vegetarian because vegetarian food is much easier to digest than non vegetarian (may be given other reason) Or For non- vegetarian food we have to kill animals this is not good thing because everybody has right to survive, etc.

Ans.48 $\frac{7}{8}$

A Student is future of a country. If a student is sincere then he/she can serve the country in a better way.

Ans.49 $\frac{27}{83}$

Pollution free environment minimize the health problems in the human being.

Ans.50

X	0	1	2	3
P(x)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Values lost by chairman –
Honesty, Integrity

Ans.51 $\frac{15}{34}$, skilled person can complete a work in better way than other person

Ans.52

X	0	1	2
P(x)	$\left(\frac{11}{20}\right)^2$	$2.\frac{9}{20}.\frac{11}{20}$	$\left(\frac{9}{20}\right)^2$

To maintain the ratio of male and female equally. This is important to consider both children are equal.

Ans.53 0.225, People in anger cannot use their presence of mind and become violent and destroy public property in riots which is indirectly their own property.

Ans.54 0.75 self studies with the help of peers is best as through it students can get the knowledge in depth of each concept. But students should be regular in school and if they feel need they could join different classes.